China's Model of Managing the Financial System*

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February 2017

Abstract

China's economic model involves active government intervention in financial markets. It relaxes/tightens market regulations and even directs asset trading with the objective to maintain market stability. We develop a theoretical framework that anchors government intervention on a mission to prevent market breakdown and the explosion of volatility caused by the reluctance of short-term investors to trade against noise traders when the risk of trading against them is sufficiently large. In the presence of realistic information frictions about unobservable asset fundamentals, our framework shows that the government can alter market dynamics by making noise in its intervention program an additional factor driving asset prices, and can divert investor attention toward acquiring information about this noise rather than fundamentals. Through this latter channel, the widely-adopted objective of government intervention to reduce asset price volatility may exacerbate, rather than improve, the information efficiency of asset prices.

*We are grateful to Darrell Duffie, Debbie Lucas, Liyan Yang, Haoxiang Zhu, and seminar participants at 2017 American Economic Association Meetings, NBER Chinese Economy Meeting, 2016 Riksbank Macroprudential Conference, the 2nd Bank of Canada-University of Toronto Conference on Chinese Economy, the 2nd China-Europe Conference on Transparency, Economic Institutions and Governance, and University of Toronto for helpful comments and suggestions.

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1 Introduction

China has experienced rapid growth in the last three decades, and has become an important part of the global economy. Its underdeveloped financial system, however, has recently been a source of great anxiety for investors and policy makers across the world. This anxiety has been driven, in part, by the turmoil in its stock markets in 2015, the sudden devaluation of its currency in 2015 that raised doubts about the government's ability to manage its exchange rate, its overheating housing markets, and its growing leverage at a national level. To fully understand these issues, it is important to systematically examine the distinct structures and features of the Chinese economy and financial system.[1]

A striking feature of the Chinese financial system is how actively China's government manages it in order to promote financial stability. The government does so through frequent policy changes, using a wide array of policy tools ranging from changes in interest rates and bank reserve requirements to stamp taxes on stock trading, suspensions and quota controls on IPO issuances, rules on mortgage rates and first payment requirements, providing public guidance through official media outlets, and direct trading in asset markets through government sponsored institutions. As potential justification for such large-scale, active interventions, China's financial markets are highly speculative,[2] and largely populated by inexperienced retail investors.[3] Its markets experience high price volatility and the highest turnover rate among major stock markets in the world. In addition, the Chinese government's paternalistic culture motivates it to view stabilizing markets and protecting retail investors as a priority.

While highly relevant for investors and policy makers, the impact of such active government intervention in asset markets is still not well understood. Even within the OECD, governments engaged in unconventional monetary policy with large-scale asset purchases during the financial crisis and subsequent recession. Understanding its tradeoffs, consequently,

[1] See, for instance, Song, Storesletten, and Zilibotti (2011) for a theoretical model of how the financial system asymmetrically provides credit to the state and non-state sectors in China.

[2] As evidence of this, Mei, Scheinkman, and Xiong (2009) illustrate how speculative trading by Chinese investors might have contributed to a systematic deviation in the prices of A and B shares issued by the same Chinese companies. The key difference between these shares is that A shares can only be held by Chinese investors, while B shares can only be held by foreign investors before 2001. Furthermore, Xiong and Yu (2011) document a spectacular bubble in Chinese warrants from 2005-2008, during which Chinese investors actively traded a set of deep out-of-money put warrants that had zero fundamental value.

[3] Retail investors in China's stock markets hold about 50% of its tradable shares and contribute to about 90% of its trading volume.

is important not only for promoting stability in the Chinese financial system, but also for navigating the post financial crisis environment, in which even governments in the OECD may be prepared to intervene after episodes of high volatility or severe market dysfunction.

We develop a conceptual framework to analyze these interventions, and, specially, government intervention through directly trading against noise traders in asset markets. To do this, we build on the standard noisy rational expectations models of asset markets with asymmetric information, such as Grossman and Stiglitz (1980) and Hellwig (1980), and their dynamic versions, such as He and Wang (1995) and Allen, Morris, and Shin (2006). In these models, noise traders create short-term price fluctuations and a group of rational investors, each acquiring a piece of private information, trade against these noise traders to provide liquidity and to speculate on their private information. Our setting includes a new player, a government, which is prepared to trade against the noise traders subject to a penalty on its trading activity.

That the asset fundamental in our setting is unobservable reflects realistic information frictions faced by investors and policy makers in the Chinese economy. To capture additional realistic features of the Chinese economy, we assume that investors are myopic to reflect the highly speculative nature of Chinese investors. Noise trading in our framework arises as a result of inexperienced retail investors in the Chinese markets, who contribute to price volatility and instability. Realistic information frictions and moral hazard also introduce unintended noise into the government's intervention, with the magnitude of this noise increasing with the intensity of government's intervention. In addition, each investor has to choose between acquiring a private signal about the asset fundamental or about this government noise before trading. This information acquisition decision reflects the fixed costs and limits to investor attention associated with acquiring information.

In accordance with the Chinese government's paternalistic culture in maintaining market stability, the government in our framework adopts a weighted objective of improving information efficiency of the asset price and reducing asset price volatility. These criteria are widely used in government intervention not only in China but also in many other countries. A conventional wisdom posits that reducing price volatility, which is easily implementable, is consistent with a more fundamental, yet difficult to implement, objective of improving the information efficiency of asset prices. This occurs because the government's trading against noise traders simultaneously reduces price volatility and improves price efficiency.

With these elements, we build up our analysis in several steps. First, we characterize a benchmark economy with perfect information, in which all investors and the government observe the fundamental. We show that, in the absence of government intervention, the asset price volatility may explode and the market may even break down when the volatility of noise trading becomes sufficiently high. This market breakdown occurs because of the myopia of investors. As investors are concerned only with the short-term return from trading the asset, their required return increases with the volatility of noise trading, and this, in turn, makes the asset price more sensitive to noise trading. As a result, the volatility of the asset price rises with noise trading volatility, and may explode when it becomes sufficiently large, which further raises the return required by investors to provide liquidity to noise traders. This feedback process can cause the market to break down because there may not exist any risk premium that can induce the investors to trade. Such a market breakdown introduces a role for the government to reduce market volatility and stabilize the market. In this environment, the government's objective of reducing price volatility is fully consistent with improving price efficiency.

We then consider an extended setting in which the asset fundamental is unobservable to both investors and the government. We first illustrate that, in the absence of government intervention, reducing price volatility introduced by noise traders is consistent with improving price efficiency. We then show that introducing government intervention can give rise to several unintended consequences. As a large player in the asset market, the government unavoidably makes the noise in its intervention an additional pricing factor in asset prices, even though it internalizes its impact on asset prices. This new pricing factor, in turn, can attract speculation by investors in the presence of informational frictions, who may choose to acquire private information about the government's noise and, as a result, be distracted from acquiring information about the fundamental. In doing so, the investors' speculation about the government's noise can further mitigate the price volatility caused by noise traders, but can exacerbate the information efficiency of the asset price.[4]

Whether investors focus on acquiring information about the asset fundamental or the

[4]There is a growing literature that explores how uncertainty about government policy introduces a non-fundamental factor into asset prices. See, for instance, Sialm (2006) and Pastor and Veronesi (2012, 2013). Our focus here is not on providing a microfoundation for this uncertainty, which we take as given, but in demonstrating how such uncertainty interacts with investor incentives to acquire information. A previous draft of this paper, which had qualitatively similar insights, microfounded the uncertainty as noise in private signals that the government receives about the asset fundamental.

government noise determines the behavior of asset prices, and can give rise to two types of equilibrium, which we label "fundamental-centric" and "government-centric", respectively. In the fundamental-centric equilibrium, investors each acquire a private signal about the fundamental and the asset price aggregates their information to partially reveal the fundamental. In this equilibrium, the government trades against both the noise traders, to minimize their price distortion, and against the investors, based on their respective private information. In the government-centric equilibrium, the investors focus on learning about the government's noise, and share a similar belief with each other and with the government about the fundamental. Consequently, they tend to trade alongside the government against the noise traders, which reinforces the government's effort to reduce price volatility and renders the government intervention more effective in reducing the price distortion of the noise traders. The reduced price volatility, however, comes at the expense of asset prices being less informative about the fundamental.

Generally speaking, whether a fundamental-centric or government-centric equilibrium appears depends on the scale of the government's intervention. There is a tendency in our economy for the market to shift from a fundamental-centric equilibrium to a government centric equilibrium as the government's intervention intensifies beyond a certain threshold, and the noise the government introduces plays a sufficiently large role in asset prices. This occurs, for instance, when the government assigns a sufficiently large weight to reducing price volatility, or when there is sufficient volatility from noise trading in the market.

Overall, our model delivers several key insights not only for government intervention in China but also more generally for intervention programs in other countries. First, it demonstrates that, even in the absence of information frictions, there can be a role for government intervention to reduce price volatility and mitigate the possibility of a market breakdown. Second, such intervention can make the government noise an additional factor in asset prices, and this additional factor may attract the speculation of investors and distract them from acquiring private information about the fundamental. This speculation, in turn, reinforces the impact of noise in the government's policy on asset prices. These two implications capture important observations about China's financial markets—speculation about government policies plays a central role in driving market dynamics and market participants pay great attention to government policies, although less so to economic fundamentals.

Finally, we also discuss how a time-inconsistency problem may arise when the government cannot pre-commit to an intervention strategy. ***.

A literature review follows. Section 2 sets up the model with perfect information, and derives the equilibria with and without government intervention. Our analysis here illustrates how a market breakdown can be avoided by the latter. Section 3 extends the setting to incorporate information frictions, derives the new equilibria under different settings without and with government intervention, and analyzes the effects of government intervention. Section 4 concludes with some additional discussion. We cover the salient features of the equilibria under different settings in the main text, while leaving more detailed descriptions of the equilibria and the key steps for deriving the equilibria in the appendix. We also provide a separate online appendix that contains all technical proofs involved in our analysis.

1.1 Related Literature

Our work contributes to the growing literature on the impact of policy uncertainty on asset prices and the macroeconomy. Pastor and Veronesi (2012, 2013) explore the asset pricing implications of uncertainty about government policy outcomes and potential changes to policy regimes. Fernandez-Villaverde et al (2013) investigates the macroeconomic consequences of uncertainty in fiscal policies. Sialm (2006) analyzes the asset pricing implications of uncertainty about investor taxation in an endowment economy, while Croce, Kung, Nguyen, and Schmid (2012) assesses its role in a production economy with recursive preferences. Croce, Nguyen, and Schmid (2012) examines the interaction between fiscal uncertainty and long-run growth when agents also face model uncertainty, and Ulrich (2013) studies how bond markets respond to Knightian uncertainty over the effectiveness of government policies. Baker, Bloom, and Davis (2015) empirically links government policy uncertainty to business cycle fluctuations. Our work extends this literature by studying policy uncertainty in the context of government interventions in financial markets.

Our work also contributes to the literature on dynamic models of asymmetric information in asset markets, which includes Wang (1994), He and Wang (1995), Allen, Morris, and Shin (2006), and Bacchetta and van Wincoop (2006, 2008). Different from these studies, our model features a large agent (i.e., the government) with price impact, in addition to a continuum of small investors, each possessing private information. In our setting, noise in the government's trading becomes an additional pricing factor, and, more interestingly, a target of speculation by the investors.

We also contribute to the literature on the financial market implications of government intervention. Gertler and Kiyotaki (2013) constructs a framework for analyzing the macroeconomic impact of large scale asset purchases by central banks. Bond and Goldstein (2015) studies the impact on information aggregation in prices when uncertain, future government intervention influences a firm's real outcomes. Boyarchenko, Lucca, and Veldkamp (2016) study the impact on information diffusion and propagation, and Treasury auction revenues, when the government can choose the form of the auction, and consequently the market structure. Cong, Grenadier, and Hu (2017) explore the information externality of government intervention in money market mutual funds in a global games framework in which investors face strategic coordination issues and intervention changes the information publicly available to them. In contrast to these studies, we focus on the incentives for information acquisition among market participants when there is uncertainty about the scope of government intervention in financial markets through large-scale asset purchases.[5]

Stein and Sundarem (2016) develops a model to analyze the communication between the Federal Reserve and the bond market. By assuming that the Fed has an objective to minimize the volatility of long-term interest rate, the model shows that a signal jamming mechanism may operate that renders the Fed ineffective in communicating information to the market and making bond prices more informative. Our model also highlights a tension between the government's objective in reducing price volatility and improving information efficiency, albeit through a different mechanism related to the information acquisition decisions of investors.

2 Market Breakdown in a Perfect-Information Model

Consider an infinite horizon economy in discrete time with infinitely many periods: $t = 0, 1, 2....$ There is a risky asset, which can be viewed as stock issued by a firm that has a stream of cash flows D_t over time:

$$D_t = \theta_t + \sigma_D \varepsilon_t^D.$$

The components θ_t is a persistent component of the fundamentals, while ε_t^D is independent and identical cashflow noise with a Gaussian distribution of $\mathcal{N}(0,1)$ and $\sigma_D > 0$ measures

[5] Similar to Goldstein and Yang (2015), prices in our setting can aggregate information about multiple fundamentals, and the fundamentals that investors care about can be different from those the government finds most relevant.

the volatility of cashflow noise.

Government policies in practice may affect asset markets through several channels. The government can directly affect the profitability of firms. We do not focus on this direct channel since we do not include an additional government cash flow term beyond $\theta_t + \varepsilon_t^D$. The literature has already studied this direct effect.[6] Instead, we intend to analyze a different channel, through which the government intervention can impact the market dynamics even when it does not directly affect the firm's cash flow. Specifically, we assume that the asset's cash flow is fully determined by an exogenous fundamental θ_t, which follows an AR(1) process:

$$\theta_t = \rho_\theta \theta_{t-1} + \sigma_\theta \varepsilon_t^\theta,$$

where $\rho_\theta \in (0,1)$ measures the persistence of the asset fundamental, $\sigma_\theta > 0$ measures the fundamental volatility, and $\varepsilon_t^\theta \sim \mathcal{N}(0,1)$ is independently and identically distributed shocks to the asset fundamental.

In this section, we assume that θ_t is **observable** to all agents in the economy. This setting serves as a benchmark for examining the role of government intervention. We will remove this assumption to make θ_t unobservable to both the government and investors in the next section to discuss how government intervention affects the investors' information acquisition.

For simplicity, suppose that there is also a riskfree asset in elastic supply that pays a constant gross interest rate $R^f > 1$. In what follows, we define R_{t+1} to be the excess payoff to holding the risky asset:

$$R_{t+1} = D_{t+1} + P_{t+1} - R^f P_t.$$

There are three types of agents in the asset market: noise traders, investors, and the government.

[6]For example, if the government faces a time-varying cost in implementing such a policy, the cost of the government policy can become an important factor in driving the stock cash flow and thus price dynamics. See Pastor and Veronesi (2012, 2013) for recent studies that explore this channel. In addition, when government policies affect the cash flow of publicly traded firms, Bond and Goldstein (2015) shows that such intervention feeds back into how market participants trade on their private information. This results in socially inefficient aggregation of private information about the unobservable fundamental θ_t into asset prices, which can impede policymaking if the government also infers relevant information about θ_t from the traded asset price in determining the scale of its intervention.

2.1 Noise Traders

It is widely observed that there is a large number of inexperienced retail investors in China's stock markets. Motivated by this observation, we assume that in each period, these inexperienced investors, whom we call noise traders, submit exogenous market orders into the asset market.[7] We denote the quantity of their orders by N_t and assume that N_t also follows an AR(1) process:

$$N_t = \rho_N N_{t-1} + \sigma_N \varepsilon_t^N,$$

where $\rho_N \in (0,1)$ measures the persistence of noise trading, $\sigma_N > 0$ measures the volatility of noise trading (or noise trader risk in this market), and $\varepsilon_t^N \sim \mathcal{N}(0,1)$ is independently and identically distributed shocks to noise traders. The presence of noise traders creates incentives for other investors to trade in the asset market. To the extent that investors may not be able to fully eliminate the market impact of noise traders, this also justifies government intervention to further dampen the impact of noise traders.

2.2 Investors' Problem

There are a continuum of investors in the market who trade the asset on each date t. We assume that these investors are myopic and live for only one period. That is, in each period a group of new investors with measure 1 join the market, replacing the group from the previous period. We index an individual investor by $i \in [0,1]$. Investor i born at date t is endowed with wealth $\bar{W}$ and has constant absolute risk aversion CARA preferences with coefficient of risk aversion γ over its next-period wealth W_{t+1}^i:

$$U_t^i = E\left[-\exp\left(-\gamma W_{t+1}^i\right) \mid \mathcal{F}_t\right].$$

It purchases X_t^i shares of the asset and invests the rest in the riskfree asset at a constant rate R^f, so that W_{t+1}^i is given by

$$W_{t+1}^i = R^f \bar{W} + X_t^i R_{t+1}.$$

The investors have symmetric, perfect information, and their expectations are all taken with respect to the full-information set $\mathcal{F}_t = \sigma\left(\{\theta_s, N_s, D_s\}_{s\leq t}\right)$ in this section. As a result

[7] This way of modeling noise trading is standard in the market microstructure literature. See Black (1986) for a classic reference. Even though we do not explicitly model their origination, we think of these random orders as emanating from fluctuations in the retail investors' sentiment and overreactions to relevant or irrelevant information.

of CARA preferences, an individual investor's trading behavior is insensitive to his initial wealth level.

The assumption of investor myopia is commonly used in dynamic models of assset markets with informational frictions for simplicity, e.g., Allen, Morris, and Shin (2006) and Bacchetta and van Wincoop (2006). In our setting, this assumption also serves to capture the speculative nature of Chinese investors, which is important for generating market breakdown when noise trader risk becomes sufficiently large.

2.3 Equilibrium without Government

To facilitate our discussion of this perfect-information setting, we first characterize the rational expectations equilibrium without government intervention. Because investors are risk-averse, the market can only bear a limited amount of noise trader risk. In what follows, we derive the equilibrium price and show formally that the market breaks down whenever the noise trader risk σ_N rises above a certain threshold. Moreover, we show that the excess return volatility in equilibrium is increasing in σ_N, and the rate of this volatility increase grows explosively as σ_N approaches the threshold.

We conjecture a linear rational expectations equilibrium and then verify that there cannot be any nonlinear equilibrium—see Appendix A for more details of the equilibrium construction. In this equilibrium, the asset price P_t is a linear function of the fundamental θ_t and the noise trader shock N_t:

$$P_t = \frac{1}{R^f - \rho_\theta}\theta_t + p_N N_t,$$

where $\frac{1}{R^f-\rho_\theta}\theta_t$ is the expected present value of cashflows from the asset. This conjecture implies that the conditional excess return variance is given by

$$Var\left(R_{t+1}|\mathcal{F}_t\right) = \sigma_D^2 + \left(\frac{1}{R^f - \rho_\theta}\right)^2 \sigma_\theta^2 + p_N^2\sigma_N^2.$$

Since all investors are symmetrically informed and have CARA utility with normally distributed payoffs, they will have identical demand for the risky asset X_t^i:

$$X_t^i = \frac{1}{\gamma}\frac{p_N\left(\rho_N - R^f\right)}{\sigma_D^2 + \left(\frac{1}{R^f-\rho_\theta}\right)^2\sigma_\theta^2 + p_N^2\sigma_N^2}N_t.$$

Each myopic investor's demand trades off the expected asset return with the return variance over the subsequent period.

Then, imposing market-clearing in the asset market

$$X_t^i = N_t,$$

leads to a quadratic equation that pins down the price coefficient p_N. Note that there exist two negative roots to p_N. Similar to Bacchetta and van Wincoop (2006, 2008), we focus on the less negative root of the two. This is sensible since as $\sigma_N \to 0$ (i.e., noise trader risk vanishes from the economy), the less negative root has $p_N \sigma_N \to 0$ (i.e., the price impact of noise traders diminishes), while the more negative root diverges. We always focus on this more stable root of the two in our analysis, hereafter.

The following proposition, with details provided in the Appendix, shows the equilibrium does not exist if σ_N is higher than a threshold:

$$\sigma_N^* = \frac{R^f - \rho_N}{2\gamma\sqrt{\sigma_D^2 + \left(\frac{1}{R^f - \rho_\theta}\right)^2 \sigma_\theta^2}}. \tag{1}$$

Proposition 1 *If the noise trader risk $\sigma_N \leq \sigma_N^*$, an equilibrium exists with $\frac{\partial(Var(R_{t+1}|\mathcal{F}_t))}{\partial \sigma_N^2} > 0$, and $\frac{\partial(Var(R_{t+1}|\mathcal{F}_t))}{\partial \sigma_N^2} \to \infty$ as $\sigma_N \to \sigma_N^*$ implying that the asset return variance is highest at $\sigma_N = \sigma_N^*$ with a value of $2\left[\sigma_D^2 + \left(\frac{1}{R^f - \rho_\theta}\right)^2 \sigma_\theta^2\right]$. If $\sigma_N > \sigma_N^*$, no equilibrium exists.*

The proposition shows that the asset return variance increases with the noise trader risk σ_N and the rate of this increase explodes as σ_N gets close to the threshold σ_N^*. Figure 1 illustrates the explosive return variance as σ_N approaches σ_N^*. Furthermore, this proposition establishes that the market breaks down when σ_N rises above σ_N^*.

Intuitively, since the investors are myopic and care only about the risk and return over the subsequent one period, they become increasingly reluctant to trade against noise traders as σ_N rises. As σ_N rises, investors would demand a higher risk premium to take on a position against noise traders, reflected in a larger coefficient p_N, which, in turn, leads to a higher asset return volatility. Through this feedback process, once σ_N gets larger than σ_N^*, the asset return volatility becomes so large that the investors are not willing to take on any position regardless of the risk premium. The dynamic setting further exacerbates this issue, since any price concession in conditional risk premia today necessary to entice investors to accommodate noise traders is, in part, offset by the similar concession needed tomorrow to entice the new generation of investors, reflected in the ρ_N term in the numerator.

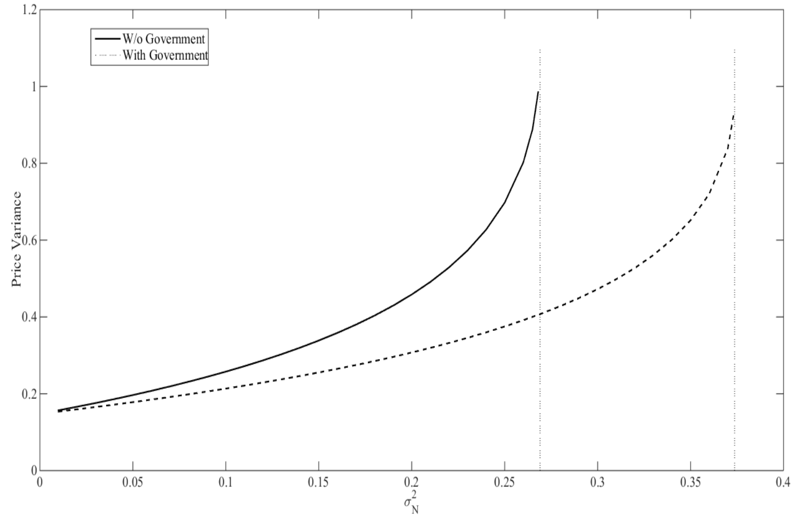

Figure 1: Asset price variance with and without government intervention as the variance of noise trading σ_N^2 increases. The solid line represents the case without government intervention, and the dashed line represents the case with government intervention at a given intensity of $\vartheta_N = 0.2$. This figure is based on the following model parameters: $\gamma = 1$, $R^f = 1.01$, $\rho_\theta = 0.75$, $\sigma_\theta^2 = 0.01$, $\sigma_D^2 = 0.08$, $\rho_N = 0$, $\vartheta_N = 0.2$.

In this setting, the myopia of investors and the price-insensitive trading of noise traders jointly lead to the market breakdown. If the investors have longer horizons, they would be willing to take on a position despite the large return volatility over the short-term, which would, in turn, stabilize the price impact of noise traders. As such, the reluctance of short-term investors to trade against noise traders is reminiscent of the classic result highlighted by De Long, et al (1990), which shows that noise traders can create their own space in asset prices in the presence of myopic arbitrageurs.

The noise traders' price-insentitive trades serve to capture market rigidity that sometimes occurs as a result of either forced fire sales or panick selling during market turmoil. If the noise traders have a price-sensitive supply curve, there would always be a market-clearing price. Nevertheless, in such a case, noise in the noise traders' supply curve would still lead to higher asset price volatility, which risk-averse investors would not be able to fully remove, and thus nevertheless motivate government intervention.[8]

[8]The presence of high price volatility in China's financial markets, which is possibly driven by the large number of retail investors and the short-termism of other investors, also makes it difficult for a financial institution to pursue a long-term investment strategy as a result of the agency problems highlighted by Shleifer and Vishny (1997). This issue, while not explicitly modeled in our setting, also motivates government intervention to reduce price volatility.

2.4 Equilibrium with Government Intervention

We now augment the baseline setting to include a government that actively intervenes in the asset market. Specifically, we assume that the government's asset trading follows a linear trading rule:

$$X_t^G = \vartheta_{N,t} N_t + \sqrt{Var\left[\vartheta_{N,t} N_t \mid \mathcal{F}_{t-1}\right]} G_t,$$

where the coefficient $\vartheta_{N,t}$ represents the government's intervention strategy in trading against the noise traders, and $\sqrt{Var\left[\vartheta_{N,t} N_t \mid \mathcal{F}_{t-1}\right]} G_t$ is an unintended noise component that arises from frictions in the intervention process, such as behavioral biases, lobbying effort, or information frictions. Specifically, $G_t = \sigma_G \varepsilon_t^G$ with $\varepsilon_t^G \sim \mathcal{N}(0,1)$ as independently and identically distributed shocks and σ_G as a volatility parameter. The magnitude of this noise component scales up with the conditional volatility of the intended intervention strategy $\sqrt{Var\left[\vartheta_{N,t} N_t \mid \mathcal{F}_{t-1}\right]}$, which is equal to $\sigma_N \vartheta_{N,t}$ with perferect information. This specification is reasonable as it is easier for frictions to affect the government intervention when the intervention strategy requires more intensive trading. Furthermore, the government can neither correct nor trade against its own noise, because the noise originates from its own system. Instead, as we will analyze later, the government can internalize the amount of noise by limiting its trading intensity.

We assume the government intervenes with a benevolent objective of stabilizing the market, which is consistent with what is often stated by the Chinese government. There are two different variations in implementing this general objective in practice: one is to minimize the deviation of asset prices from fundamentals; the other is simply to minimize asset price volatility. Each of these two objectives has its own appeal and can be micro-founded under suitable assumptions. The former is consistent with making asset prices more informative and thus more efficient in guiding resource allocation in the economy, while the latter is consistent with reducing destabilizing effects of asset price volatility on leveraged investors and firms. These two objectives are closely related and are often treated as consistent with each other in government intervention. This is because by reducing the price impact of noise traders, government intervention reduces both asset price deviation from fundamentals and price volatility. As in practice price volatility is easy to measure while asset price deviation from fundamentals is difficult to detect, reducing asset price volatility is a more appealing objective of government intervention in practice and indeed is widely adopted.[9] Our analysis intends to compare these two objectives and show that they may deviate from each other in the presence of information frictions.

Specifically, we adopt the following general specification to represent the government's myopic preference for intervention in the asset market at date t:

$$U_t^G = \min_{\vartheta_{N,t}} \ \gamma_\sigma Var\left[\Delta P_t\left(\vartheta_{N,t}\right)|\mathcal{F}_t\ \right] + \gamma_\theta Var\left[P_t\left(\vartheta_{N,t}\right) - \frac{1}{R^f-\rho_\theta}\theta_{t+1}\ |\mathcal{F}_t\right] .$$

The first term $\gamma_\sigma Var\left[\Delta P_t\left(\vartheta_{N,t}\right)|\mathcal{F}_t\ \right]$ captures a goal to minimize the conditional asset price variance, with the coefficient $\gamma_\sigma \geq 0$ measuring the government's aversion to price volatility. The second term $\gamma_\theta Var\left[P_t\left(\vartheta_{N,t}\right) - \frac{1}{R^f-\rho_\theta}\theta_{t+1}\ |\mathcal{F}_t\right]$ captures a goal to reduce price inefficiency, with the coefficient $\gamma_\theta \geq 0$ measuring the government's aversion to the conditional variance of the asset price deviation from the asset's fundamental value $\frac{1}{R^f-\rho_\theta}\theta_{t+1}$.

There are several notable points about this objective function. First, since the two components in the government's objective are both second moments, we shall consider only stationary policies, $\vartheta_{N,t} = \vartheta_N$. Second, this objective function can be scaled up or down by any positive constant without affecting the government's optimal choice. Third, this objective function does not contain any cost for government intervention. Interestingly, despite the absence of any intervention cost, there is an interior optimum to the government's intervention strategy, because the government internalizes the amount of noise its intervention brings to the market.

As the government trades alongside investors to accommodate the asset supply of noise traders, the market-clearing condition $\int_0^1 X_t^i di + X_t^G = N_t$ implies the following equilibrium asset price function with the government noise as an additional factor:

$$P_t = \frac{1}{R^f-\rho_\theta}\theta_t + p_N N_t + P_g G_t.$$

In Appendix A, we show that a market equilibrium exists when

$$\sigma_N < \frac{1}{\left(1-\vartheta_N\right)\sqrt{\left(1+\left(\frac{\rho_N-R^f}{R^f}\right)^2\left(\frac{\vartheta_N}{1-\vartheta_N}\right)^2\sigma_G^2\right)}}\sigma_N^* \tag{2}$$

where σ_N^* is given in equation (1). The more aggressively the government trades to accommodate noise trading, the closer is ϑ^N to 1, and the slacker is the equilibrium existence

[9] For example, a recent study by Stein and Sunderam (2016) adopts reducing volatility of long-term interest rate as the objective of the Federal Reserve Board in managing its monetary policy in the U.S.

condition compared to the case without the government intervention (i.e., $\vartheta_N = 0$.) This is shown in Figure 1, which depicts the shift in the market breakdown upper-bound and also the reduced asset price volatility before σ_N reaches the upper-bound.

To the extent that the asset price variance and the variance of the asset price from its fundamental value both explode when the market breaks down, the government's aversion to any of these outcomes would motivate the government to choose a sufficiently large ϑ_N so that the condition in (2) is satisfied. Thus, a market equilibrium always prevails. Furthermore, the government objective of improving price efficiency is qualitatively consistent with an alternative of reducing price volatility.

Taken together, government intervention in asset markets can help to ensure market stability, especially during times of extreme market dysfunction, when noise trader risk is high. With informational frictions, however, the intervention to stabilize asset prices has additional effects on market dynamics, which we illustrate in the next section.

3 An Extended Model with Information Frictions

We now extend the model to introduce realistic information frictions that investors and the government face in financial markets, while keeping the market structure and the trading preferences of investors and the government similar. Specifically, we assume that the asset fundamental θ_t and noise trading N_t are both unobservable to all agents in the economy. This extended model allows us to analyze how government intervention interacts with both trading and information acquisition of investors, ultimately affecting the information efficiency of asset prices.

Furthermore, for simplification, we assume that the noise in government trading G_t is publicly observable at date t albeit not before t.[10] As the government noise affects the asset price in equilibrium, investors have an incentive to acquire information about the next-period's government noise, and this incentive may be even greater than the incentive to acquire information about the asset fundamental. Indeed, our model shows that while government intervention dampens price volatility when this occurs, it may exacerbate rather than improve the information efficiency of the asset price.

[10] In an earlier draft of the paper, we have analyzed the case with G_t being unobservable even after t. The results are qualitatively similiar to our current setting, although the analysis is substantially more complex.

3.1 Information and Equilibrium

This subsection describes the information structure of the economy with information frictions, building on the primitives of the model and the preferences of the various economic agents in the perfect information setting.

3.1.1 Public Market Information

All market participants observe the full history of all public information, which includes all past dividends, asset prices, and government noise:

$$\mathcal{F}_t^M = \{D_s, P_s, G_s\}_{s \leq t},$$

which we will hereafter refer to as the "market" information set. We define

$$\hat{\theta}_{t+1}^M = E\left[\theta_{t+1} \mid \mathcal{F}_t^M\right]$$

as the conditional expectation of θ_{t+1} with respect to $\mathcal{F}_t^M$. The government needs to trade against the noise trading based on its conditional expectation of N_t. Without any private information, its expectation of N_t is equal to the expectation conditional on $\mathcal{F}_t^M$. At the risk of abusing notation, we define

$$\hat{N}_t^M = E\left[N_t \mid \mathcal{F}_t^M\right].$$

Importantly, $\hat{N}_t^M$ represents expectation of the current-period N_t rather than N_{t+1}. Furthermore, we define

$$\hat{G}_{t+1}^M = E\left[G_{t+1} \mid \mathcal{F}_t^M\right]$$

as the market's conditional expectation of the next-period G_{t+1}. These three belief variables, $\hat{\theta}_{t+1}^M$, $\hat{N}_t^M$, and $\hat{G}_{t+1}^M$, are time-t expectations of θ_{t+1}, N_t, and G_{t+1}, respectively. Together with the publicly observed current-period G_t, they summarize the public information at time t regarding the aggregate state of the market. We collect these variables as a state vector

$$\Psi_t = \left[\begin{array}{cccc} \hat{\theta}_{t+1}^M & \hat{N}_t^M & G_t & \hat{G}_{t+1}^M \end{array}\right].$$

3.1.2 Government

At date t, the government's information set contains only the publicly available information $\mathcal{F}_t^M$.[11] Like the previous setting, we assume that the government has an intervention

[11] In a previous draft, we adopted an alternative setting in which the government possesses private signals about the fundamental. This private information causes the government to hold different beliefs about

program, which is instituted to trade against the noise trading in the market based on its conditional expectation $\hat{N}_t^M$:

$$X_t^G = \vartheta_{\hat{N},t}\hat{N}_t^M + \sqrt{Var\left[\vartheta_{\hat{N},t}\hat{N}_t^M \mid \mathcal{F}_{t-1}^M\right]}G_t. \tag{3}$$

Furthermore, the government has a similar myopic objective as before in choosing its intervention strategy at date t:

$$U_t^G = \min_{\vartheta_{\hat{N},t}} \gamma_\sigma Var\left[P_t\left(\vartheta_{\hat{N},t}\right) \mid \mathcal{F}_{t-1}^M\right] + \gamma_\theta Var\left[P_t\left(\vartheta_{\hat{N},t}\right) - \frac{1}{R^f - \rho_\theta}\theta_{t+1} \mid \mathcal{F}_{t-1}^M\right]. \tag{4}$$

As both of these variance terms are conditional on the government's information set $\mathcal{F}_{t-1}^M$, one can view the government as choosing its intervention strategy $\vartheta_{\hat{N},t}$ at date t before the investors observe any private information. The two terms in the government's objective are all second moments, which are deterministic in our Gaussian setting, thus they are all computable despite the non-nesting of information sets between the government and investors.

In maximizing (4), the government recognizes that its trading directly affects the asset price $P_t\left(\vartheta_{\hat{N},t}\right)$. In other words, the government is a large player that internalizes its impact on the asset market. It is fully aware of how its trading impacts the asset price through market clearing and, through this channel, the informativeness of the asset price as a signal about θ_t and G_t. This informativeness of the asset price impacts not only the government's ability to learn from the asset price, but also that of the investors. Moreover, the government also internalizes its impact on the investors' information acquisition.

3.1.3 Investors

In each period, the investors face uncertainty in the asset fundamental, the noise trading, and the government noise. Specifically, at date t, each investor can choose to acquire a private signal either about the next-period asset fundamental θ_{t+1} or about the next-period government noise G_{t+1}. We denote the investor's choice as $a_t^i \in \{0, 1\}$, with 0 representing the choice of a fundamental signal and 1 the choice of a signal about the government noise.

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the fundamental and noise trading from investors and, more importantly, makes the government's trading not fully observable to the investors. Through this latter channel, the noise in the government's signals endogenizes the government noise G_t. Such a structure substantially complicates the analysis, however, by introducing a double learning problem for the investors to acquire information about the government's belief, which is itself the outcome of a learning process. It is reassuring that this more elaborate setting gives similar results as our current setting with exogeneous government noise.

When the investor chooses $a_t^i = 1$, the fundamental signal is

$$s_t^i = \theta_{t+1} + \left[a_t^i \tau_s\right]^{-1/2} \varepsilon_t^{s,i},$$

where $\varepsilon_t^{s,i} \sim \mathcal{N}(0,1)$ is signal noise, independent of all other random variables in the setting, and τ_s represents the precision of the signal if chosen. When the investor chooses $a_t^i = 0$, the government signal is

$$g_t^i = G_{t+1} + \left[\left(1 - a_t^i\right) \tau_g\right]^{-1/2} \varepsilon_t^{g,i},$$

where $\varepsilon_t^{g,i} \sim \mathcal{N}(0,1)$ is signal noise, independent of all other random variables in the setting, and τ_g represents the precision of the signal if chosen. These signals allow the investor to better predict the next-period asset return by forming more precise beliefs about θ_{t+1} and G_{t+1}. Motivated by limited investor attention and realistic fixed cost in information acquisition, we assume that each investor needs to choose one and only one of these two signals.[12]

At date t, each investor first makes his information acquisition choice a_t^i based on the public information set $\mathcal{F}_{t-1}^M$ from the previous period. After receiving his private information $a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i$ and the public information D_t, P_t, and G_t released during the period, the investor chooses his asset position X_t^i to maximize his expected utility over his wealth at $t+1$:

$$U_t^i = \max_{a_t^i \in \{0,1\}} E\left[\max_{X_t^i} E\left[-\exp\left(-\gamma W_{t+1}^i\right) \mid \mathcal{F}_t^i\right] \mid \mathcal{F}_{t-1}^M\right],$$

where the investor's full information set $\mathcal{F}_t^i$ is

$$\mathcal{F}_t^i = \mathcal{F}_t^M \vee \left\{a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i\right\}.$$

The investor's objective guarantees sequential rationality of his information acquisition and trading decisions. Given his beliefs about how he will trade at date t, the investor chooses what information to acquire based on public information up to $t-1$, and then chooses his trading strategy.

3.1.4 Noisy Rational Expectations Equilibrium

Market clearing of the asset market requires that the net demand from the investors and the government equals the supply of the noise traders at each date t: $\int_0^1 X_t^i di + X_t^G = N_t$.

[12] Generally speaking, the investors may also acquire private information about noise trading, rather than asset fundamental and government noise. Introducing such a third type of private information complicates the analysis without a particular gain in insight. In our current setting, each investor can indirectly infer the value of noise trading through the publicly observed asset price.

We assume that the supply of riskless debt is elastic, and therefore the credit market clears automatically.

We also assume that the investors and the government have an initial prior with Gaussian distributions at $t = 0$: $(\theta_0, N_0) \sim \mathcal{N}\left(\left(\bar{\theta}, \bar{N}\right), \Sigma_0\right)$, where $\Sigma_0 = \begin{bmatrix} \Sigma_0^\theta & 0 \\ 0 & \Sigma_0^N \end{bmatrix}$. Note that the variables in both $\mathcal{F}_t^M$ and $\mathcal{F}_t^i$ all have Gaussian distributions. Thus, conditional beliefs of the investors and the government about θ_t and N_t under any of the information sets are always Gaussian. Furthermore, the variances of these conditional beliefs follow deterministic dynamics over time and will converge to their respective steady-state levels at exponential rates. Throughout our analysis, we will focus on steady-state equilibria, in which the belief variances of the government and investors have reached their respective steady-state levels and their policies are time homogeneous.

At time t, a Noisy Rational Expectations Equilibrium is a list of policy functions, $a^i(\Psi_{t-1})$, $X^i(\Psi_t, a_t^i s_t^i + (1 - a_t^i) g_t^i, P_t)$ and $X^G(\Psi_t)$, and a price function $P(\Psi_t, \theta_t, N_t, G_{t+1})$, which jointly satisfy the following:

- Government Optimization: Before the investors choose their optimal information acquisition policies $\{a_t^i\}_i$ and optimal trading policies $\{X_t^i\}_i$, the government chooses its intervention policy $X^G(\Psi_t) = \vartheta_{\hat{N},t}\hat{N}_t^M + \sqrt{Var\left[\vartheta_{\hat{N},t}\hat{N}_t^M \mid \mathcal{F}_{t-1}^M, \{a_t^i\}_i\right]}G_t$ with $\vartheta_{\hat{N},t}$ chosen based on its ex ante information set $\mathcal{F}_{t-1}^M$ to maximize its objective, taking into account the impact of this choice on the investors' information acquisition and trading strategies.

- Investor Optimization: An individual investor i takes as given the government's intervention strategy to make his information acquisition choice $a_t^i = a^i(\Psi_{t-1})$ based on his ex ante information set $\mathcal{F}_{t-1}^M$ and then makes his investment choice $X^i(\Psi_t, a_t^i s_t^i + (1 - a_t^i) g_t^i, P_t)$ based on other investors' information acquisition choices $\left\{a_t^{-i}\right\}_{-i}$ and his full information set $\mathcal{F}_t^i$.

- Market Clearing:

$$\int_0^1 X^i\left(\Psi_t, a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i, P_t\right) di + X^G(\Psi_t) = N_t.$$

- Consistency: investor i and the government form their expectations about θ_{t+1} and N_t based on their information sets $\mathcal{F}_t^i$ and $\mathcal{F}_t^M$, respectively, according to Bayes' Rule.

In the main part of our analysis, we assume that the government can commit to an intervention strategy, as defined above. We also discuss a time-inconsistency problem if the government cannot commit in Section 3.5.

3.2 A Benchmark without Government Intervention

Before we analyze the effects of government intervention, we first describe a benchmark equilibrium in the absence of the government. In particular, we show that the market breaks down faster than in the perfect information case, as the noise trader risk rises.

We arrive at this benchmark by letting $\gamma_\sigma = \gamma_\theta = 0$. In this case, the government would not intervene at all. Consequently, the investors would all acquire information about the asset fundamental. With each investor possessing a private signal about the asset fundamental, the setting in each period resembles that of Hellwig (1980). We systematically derive the market equilibrium in Appendix B. In this subsection we highlight a few key steps for deriving the equilibrium, as we will also follow the same steps, albeit more complex, in deriving the equilibrium with government intervention.

First, $\hat{\theta}_{t+1}^M$, the conditional expectation of the fundamental based on the public market information set $\mathcal{F}_t^M$, represents an anchor of the fundamental before each investor conditions on his private signal. Second, after an individual investor i receives his private signal s_t^i, his conditional belief is updated to

$$\hat{\theta}_{t+1}^i = E\left[\theta_{t+1}|\mathcal{F}_t^i\right] = E\left[\theta_{t+1}|\mathcal{F}_t^M \vee s_t^i\right] = \hat{\theta}_{t+1}^M + \frac{Cov\left[\theta_{t+1}, s_t^i|\mathcal{F}_t^M\right]}{Var\left[s_t^i|\mathcal{F}_t^M\right]}\left(s_t^i - \hat{\theta}_{t+1}^M\right).$$

Third, in a linear asset market equilibrium, the investor's expected excess asset return is a linear function that increases with $\hat{\theta}_{t+1}^i$ and decreases with the asset price P_t. Fourth, the investor's preference implies that his optimal asset position linearly increases with his expected excess asset return and is thus a linear function that increases with $s_t^i - \hat{\theta}_{t+1}^M$ and decreases with P_t. Fifth, given the symmetry among all investors, their aggregate asset position is a linear function, increasing with their aggregate signal $\int_0^1 \left(s_t^i - \hat{\theta}_{t+1}^M\right) di$, which is exactly $\theta_{t+1} - \hat{\theta}_{t+1}^M$ according to the Law of Large Numbers, and decreasing with P_t.

Finally, the market clearing condition of equating the investors' aggregate asset position to the supply of noise traders leads to the equilibrium asset price:

$$P_t = \frac{1}{R^f - \rho_\theta}\hat{\theta}_{t+1}^M + p_\theta\left(\theta_{t+1} - \hat{\theta}_{t+1}^M\right) + p_N N_t, \tag{5}$$

where the first component $\frac{1}{R^f - \rho_\theta}\hat{\theta}^M_{t+1}$ is the expected asset fundamental based on the market information, and the second component represents information aggregation through the investors' trading, and the third component represents the price impact of noise traders. The presence of noise trading prevents the asset price from fully revealing the asset fundamental θ_{t+1}, while the investors' trading mitigates the price impact of noise trading. The weights of these two components, p_θ and p_N, are endogenously determined in the equilibrium by various model parameters, such as the investors' risk aversion, signal precision and the noise trader risk. We derive these coefficients in the appendix, and demonstrate that p_θ is always below $\frac{1}{R^f - \rho_\theta}$, its value in the perfect-information setting.

Importantly, the asset price P_t is a key source of market information in determining $\hat{\theta}^M_{t+1}$ in the first step. We start by conjecturing the linear price function in (5) and carrying the unknown coefficients p_θ and p_N through the aforementioned steps until we solve them through the market clearing condition in the final step. This benchmark equilibrium shows that when the investors each acquire a private signal about the fundamental, the equilibrium asset price aggregates their private signals and partially reveals the fundamental.

Figure 2 illustrates this benchmark equilibrium. Panel A compares the asset price variance $Var\left[P_t \mid \mathcal{F}^M_{t-1}\right]$ in the presence and absence of information frictions by varying the noise trader risk σ^2_N. As in the full information case, a market breakdown can still occur, and it does so for an even lower threshold in noise trader risk. This earlier breakdown, which is caused by the larger risk that investors have to take in the presence of information frictions, holds for many different constellations of model parameters we have examined.

Panel B of Figure 2 shows that in the presence of information frictions the conditional variance of the price deviation from its fundamental value $Var\left[P_t - \frac{1}{R^f - \rho_\theta}\theta_{t+1} \mid \mathcal{F}^M_{t-1}\right]$ and the conditional asset price variance $Var\left[P_t \mid \mathcal{F}^M_{t-1}\right]$ are both monotonically increasing with noise trader risk σ^2_N. This pattern is consistent with a conventional wisdom that in trading against noise traders, the government's objective in improving information efficiency of asset prices (i.e., reducing price deviation from asset fundamentals) is consistent with reducing price volatility. As we will show later, however, this conventional wisdom may not hold when investors can choose to acquire information about the asset fundamental or noise in the government intervention.

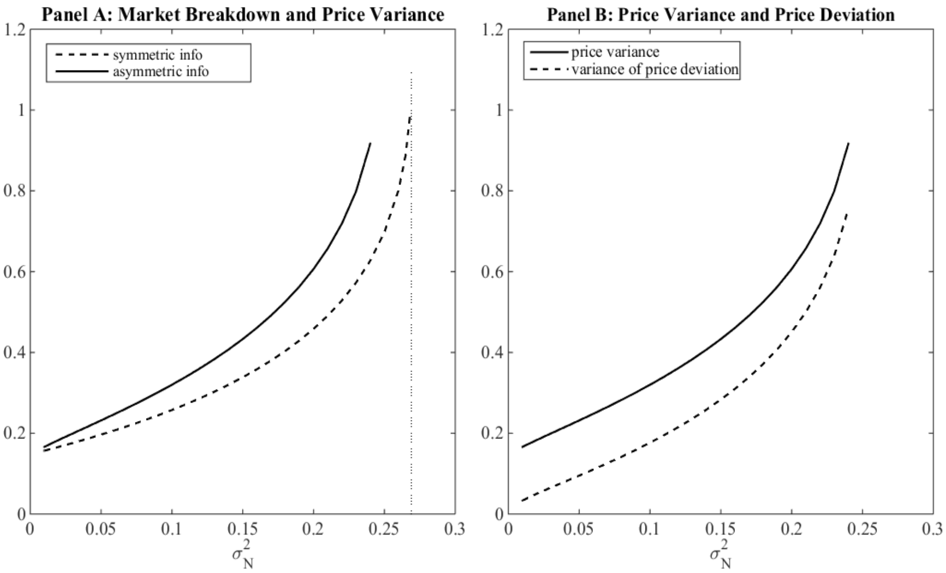

Figure 2: The benchmark equilibrium without government intervention across different values of noise trader risk. Panel A depicts asset price variance $Var\left[P_t \mid \mathcal{F}_{t-1}^M\right]$ for the symmetric information case in the dashed line and for the asymmetric information case in the solid line. Panel B depicts, for the asymmetric information case, asset price variance in the solid line and the conditional variance of price deviation from the fundamental $Var\left[P_t - \frac{1}{R^f - \rho_\theta}\theta_{t+1} \mid \mathcal{F}_t^M\right]$ in the dashed line. This figure is based on the following model parameters: $\gamma = 1$, $R^f = 1.01$, $\rho_\theta = 0.75$, $\sigma_\theta^2 = 0.01$, $\sigma_D^2 = 0.08$, $\rho_N = 0$, $\tau_s = 500$.

3.3 Equilibrium with Government Intervention

We now analyze the main setting with the government trading along with the investors. As noted previously, with the government intervention introducing noise G_t into the equilibrium asset price as an additional factor, each investor faces a choice at date t in whether to acquire private information about either the next-period fundamental θ_{t+1} or government noise G_{t+1}. When all investors choose to acquire information about the government noise, the asset price does not aggregate any private information about θ_{t+1} but rather brings the next-period government noise into the current-period asset price. This outcome may thus compromise the information efficiency of the asset price.

The derivation of the equilibrium follows similar steps as deriving the benchmark equilibrium in the previous subsection, albeit with additional complexity as a result of the government's intervention and the investors' speculation of the noise in government intervention. To simplify the presentation, we describe key elements of the equilibrium in this subsection in order to convey the key economic mechanism of the model. We provide the complete steps and formulas in Appendix C.

3.3.1 Price Conjecture and Equilibrium Beliefs

We begin by conjecturing a linear asset price function:

$$P_t = \frac{1}{R^f - \rho_\theta}\hat{\theta}^M_{t+1} + p_g G_t + p_{\hat{G}}\hat{G}^M_{t+1} + p_\theta\left(\theta_{t+1} - \hat{\theta}^M_{t+1}\right) + p_G\left(G_{t+1} - \hat{G}^M_{t+1}\right) + p_N N_t. \quad (6)$$

The first term $\frac{1}{R^f-\rho_\theta}\hat{\theta}^M_{t+1}$ is the expected asset fundamental conditional on the market information $\mathcal{F}^M_t$ at date t, the term $p_g G_t$ reflects the noise introduced by the government into the asset demand in the current period, while the term $p_{\hat{G}}\hat{G}^M_{t+1}$ reflects the market expectation of the government noise in the next period. These three pieces serve as anchors in the asset price based on the public information. The fourth term $p_\theta\left(\theta_{t+1} - \hat{\theta}^M_{t+1}\right)$ captures the fundamental information aggregated through the investors' trading, similar to the benchmark without the government intervention. Note that if all investors choose to acquire information about the next-period government noise, rather than the asset fundamental, the coefficient of this term p_θ would be zero. The fifth term $p_G\left(G_{t+1} - \hat{G}^M_{t+1}\right)$ captures the investors' private information about the next-period government noise aggregated through their trading.[13] The final term $p_N N_t$ represents the price impact of noise trading.[14]

As before, the market belief $\hat{\theta}^M_{t+1}$ incorporates learning from the publicly observed dividend flow and asset price. As the equilibrium asset price now contains the government noise G_t in the current period, which is publicly observed, and G_{t+1} in the next period, which is not yet observable to the public, learning from asset prices is more complex and depends on the equilibrium weights of the asset price on the different factors in equation (6). In this subsection, we take the market belief $\hat{\theta}^M_{t+1}$ as given and discuss how private information causes the beliefs of individual investors to deviate from the market belief. Since the government does not have any private information, its posterior belief is the market belief, which we systematically present in Appendix C.

An individual investor not only needs to infer the asset fundamental θ_{t+1} but also the government noise G_{t+1}. As each individual investor has a piece of private signal $a^i_t s^i_t + (1 - a^i_t)$ g^i_t, his learning process simply requires adding this additional signal to the market beliefs.

[13] Notice that there is no need to incorporate a term related to investors' cross-beliefs about θ_{t+1} or G_{t+1} because $\int_0^1 a^i_t s^i_t di = \theta_{t+1}$ and $\int_0^1 \left(1 - a^i_t\right) g^i_t di = G_{t+1}$ by the Law of Large Numbers.

[14] This conjectured functional form is not unique because the market's beliefs about θ_t, N_t, and G_{t+1} are endogenous, correlated objects ex-post after observing prices. We choose this representation for expositional convenience and clarity.

We summarize the filtering process through the updating equation as

$$\begin{aligned}\begin{bmatrix}\hat{\theta}^i_{t+1}\\ \hat{G}^i_{t+1}\end{bmatrix} &= \begin{bmatrix}\hat{\theta}^M_{t+1}\\ \hat{G}^M_{t+1}\end{bmatrix} + Cov\left\{\begin{bmatrix}\theta_{t+1}\\ G_{t+1}\end{bmatrix}, a^i_t s^i_t + \left(1-a^i_t\right) g^i_t \middle| \ \mathcal{F}^M_t\right\}\\ &\quad \cdot Var\left\{ a^i_t s^i_t + \left(1-a^i_t\right) g^i_t \middle| \ \mathcal{F}^M_t\right\}^{-1}\left[a^i_t\left(s^i_t - \hat{\theta}^M_{t+1}\right) + \left(1-a^i_t\right)\left(g^i_t - \hat{G}^M_{t+1}\right)\right].\end{aligned}$$

The variance and co-variance in this expression depend on various endogenous subjects such as the informativeness of the equilibrium asset price and the precision of the market beliefs, and are fully derived in Appendix C. This expression makes clear that the investor's private signal helps him to infer the asset fundamental and the government noise in the next period, both of which impact the asset return.

The linear relationship between the investor's conditional expectations and private signals, s^i_t and g^i_t, also offers convenience in imposing market clearing. Since the noise in investors' private signals satisfies a weak Law of Large Numbers, $\int_\chi \varepsilon^{s,i}_t di = \int_\chi \varepsilon^{g,i}_t di = 0$ over an arbitrary subset of the unit interval χ, aggregating the investors' signals, s^i_t and g^i_t, will reveal both of the underlying variables θ_{t+1} and G_{t+1}. However, market clearing also includes the position of noise traders, which prevents the asset price from fully revealing these variables.

3.3.2 Investment and Information Acquisition Policies

We now examine the optimal policies of an individual investor i at date t, who takes the intervention policy of the government as given. To derive his optimal investment policy, it is convenient to decompose the expected excess return from the asset based on his information set relative to the public market information set. We can update $E\left[R_{t+1} \mid \mathcal{F}^i_t\right]$ from $E\left[R_{t+1} \mid \mathcal{F}^M_t\right]$ by the Bayes' Rule according to

$$\begin{aligned} & E\left[R_{t+1} \mid \mathcal{F}^i_t\right] = E\left[R_{t+1} \mid \mathcal{F}^M_t \vee a^i_t s^i_t + \left(1-a^i_t\right) g^i_t\right]\\ = \ & E\left[R_{t+1} \mid \mathcal{F}^M_t\right] + \frac{Cov\left[R_{t+1}, a^i_t s^i_t + (1-a^i_t)\, g^i_t \mid \ \mathcal{F}^M_t\right]}{Var\left[a^i_t s^i_t + (1-a^i_t)\, g^i_t \mid \ \mathcal{F}^M_t\right]}\\ & \times\left[a^i_t\left(s^i_t - \hat{\theta}^M_{t+1}\right) + \left(1-a^i_t\right)\left(g^i_t - \hat{G}^M_{t+1}\right)\right].\end{aligned}$$

This expression shows that the investor's private information through either s^i_t or g^i_t can help him in better predicting the excess asset return relative to the market information. This is because by using s^i_t and g^i_t to form better predictions of θ_{t+1} and G_{t+1}, the investor can better predict the asset return in the subsequent period.

Given the investor's myopic CARA preferences, his demand for the asset is

$$X^i = \frac{1}{\gamma} \frac{E\left[R_{t+1} \mid \mathcal{F}_t^i\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}. \tag{7}$$

In choosing whether to acquire either s_t^i or g_t^i at date t, the investor maximizes his expected utility based on the ex-ante market information:

$$E\left[U_t^i \mid \mathcal{F}_{t-1}^M\right] = \max_{a_t^i \in \{0,1\}} -E\left\{E\left[\exp\left(-\gamma R^f \bar{W} - \frac{1}{2}\frac{E\left[R_{t+1} \mid \mathcal{F}_t^i\right]^2}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}\right) \middle| \mathcal{F}_t^M\right] \middle| \mathcal{F}_{t-1}^M\right\}.$$

This expected utility has already incorporated the investor's optimal trading strategy in (7). We have also applied the Law of Iterated Expectations to emphasize that by first taking expectations with respect to the date-t market information $\mathcal{F}_t^M$, we can arrive at a tractable characterization of each investor's information acquisition decision.

Note that the investor's expected CARA utility in our Gaussian framework is fully determined by the second moment of the return distribution conditional on his information set $\mathcal{F}_t^i$. This nice feature allows us to simplify his information acquisition choice to

$$a_t^i = \arg\max_{a_t^i \in \{0,1\}} -Var\left[\Delta P_{t+1} | \mathcal{F}_t^M, a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i\right].$$

This objective involves only the conditional price change variance, which is stationary in the steady-state equilibria that we consider. Thus, the information acquisition choice faced by each individual investor is time-invariant. Intuitively, given the Gaussian price distribution and exponential utility for the investors, the benefit of having more precise private information lies with reducing uncertainty over the excess asset return. By noting that

$$Var\left[R_{t+1} \mid \mathcal{F}_t^M, a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i\right] = Var\left[R_{t+1} \mid \mathcal{F}_t^M\right] - \frac{Cov\left[R_{t+1}, a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i \mid \mathcal{F}_t^M\right]^2}{Var\left[a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i \mid \mathcal{F}_t^M\right]},$$

we arrive at the following proposition, which corresponds to Proposition A6 in Appendix C.

Proposition 2 *At date t, an investor i chooses to acquire information about the next-period fundamental θ_{t+1} (i.e., $a_t^i = 1$) if $\frac{Cov\left[R_{t+1}, g_t^i \mid \mathcal{F}_t^M\right]^2}{Var\left[g_t^i \mid \mathcal{F}_t^M\right]} < \frac{Cov\left[R_{t+1}, s_t^i \mid \mathcal{F}_t^M\right]^2}{Var\left[s_t^i \mid \mathcal{F}_t^M\right]}$, or about the next-period government noise G_{t+1} (i.e., $a_t^i = 0$) if $\frac{Cov\left[R_{t+1}, g_t^i \mid \mathcal{F}_t^M\right]^2}{Var\left[g_t^i \mid \mathcal{F}_t^M\right]} > \frac{Cov\left[R_{t+1}, s_t^i \mid \mathcal{F}_t^M\right]^2}{Var\left[s_t^i \mid \mathcal{F}_t^M\right]}$, or be indifferent between these two choices otherwise.*

The investor chooses his signal to maximize his informational advantage over the public information set when trading. Proposition 2 states that this objective is equivalent to choosing the signal that reduces more the conditional variance of the excess asset return, taking as given the precision of the market's information. Interestingly, this proposition shows that the investor may choose to acquire the signal on the government noise over the signal on the asset fundamental. This is because the government noise affects the asset return when the investor sells his asset holding on the next date. As a result, the more the government noise covaries with the unpredictable component of the asset return from the market information set, the more valuable the signal about the government noise is to the investor.

The choice of an individual investor to acquire information about the government noise rather than the asset fundamental introduces an externality for the overall market. When investors devote their limited attention to do so, less information about the asset fundamental is imputed into the asset price, which causes the asset price to be a poorer signal about the asset fundamental. In addition, as investors devote attention to learning about G_{t+1}, the asset price will aggregate more of the investors' private information about G_{t+1}, causing the next-period government noise to impact the current-period asset price. In this sense, the investors' speculation of government noise may exacerbate its impact on asset prices.

3.3.3 Government Policy and Market Equilibrium

We now turn to the problem faced by the government at date t. The government chooses the coefficient $\vartheta_{\hat{N},t}$ in its linear intervention policy specified in (3) to maximize its objective in (4). Note that in the steady state, each term in the objective represents a second moment that is stationary, the government's intervention policy should also be stationary. Thus, we consider only time-invariant policy conefficient $\vartheta_{\hat{N}}$. In choosing its intervention policy, the government fully internalizes the impact of its intervention on the equilibrium asset price $P\left(\vartheta_{\hat{N}}\right)$, which includes the direct impact of its trading and its impact on the investors' information acquisition policies $\{a_t^i\}_i$.

Given the investors' optimal information acquisition and trading strategies and the government's intervention strategy, we have the following market clearing condition:

$$\begin{aligned} N_t &= \vartheta_{\hat{N}}\hat{N}_t^M + \sqrt{Var\left[\vartheta_{\hat{N}}\hat{N}_t^M \mid \mathcal{F}_{t-1}^M\right]}G_t + \int \frac{a_t^i}{\gamma}\frac{E\left[R_{t+1} \mid \mathcal{F}_t^M, s_t^i\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^M, s_t^i\right]}di \\ &\quad + \int \frac{1-a_t^i}{\gamma}\frac{E\left[R_{t+1} \mid \mathcal{F}_t^M, g_t^i\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^M, g_t^i\right]}di. \end{aligned}$$

The weak Law of Large Numbers implies that aggregating the investors' asset positions will partially reveal their private information about θ_{t+1} if $a_t^i = 1$ and G_{t+1} if $a_t^i = 0$. By matching the coefficients of all the terms on both sides of this equation, we obtain a set of equations to determine the coefficients of the conjectured equilibrium price function in (6).

While we have an analytical expression for the asset market equilibria given the government's trading policy, solving the government's optimal intervention policy requires a numerical exercise to maximize its objective. As such, we rely on numerical analysis to examine the equilibrium. As we discussed in the previous section, because the government is not concerned about bearing risk in its intervention, it will always prevent any market breakdown even in the presence of information frictions, and regardless of whether the government's objective is to improve information efficiency of asset price or to reduce asset price volatility. Our numerical analysis indeed confirms the existence of equilibrium across all of the sets of model parameters that we have examined.

While an equilibrium always exists, there can exist several types of equilibria.

- **Fundamental-centric equilibrium**. When all investors choose to acquire information about the asset fundamental, the asset price aggregates the investors' private information and partially reflects the asset fundmental, and does not reflect the next-period government noise. As a result, the asset price takes a particular form of

$$P_t = \frac{1}{R^f - \rho_\theta}\hat{\theta}_{t+1}^M + p_g G_t + p_\theta\left(\theta_{t+1} - \hat{\theta}_{t+1}^M\right) + p_N N_t,$$

  which is different from the general asset price specification in (6) in that the terms $p_{\hat{G}}\hat{G}_{t+1}^M$ and $p_G\left(G_{t+1} - \hat{G}_{t+1}^M\right)$ do not appear.

- **Government-centric equilibrium**. When all investors choose to acquire information about the next-period government noise, the asset price partially reflects the next-period government noise but not the asset fundamental:

$$P_t = \frac{1}{R^f - \rho_\theta}\hat{\theta}_{t+1}^M + p_g G_t + p_{\hat{G}}\hat{G}_{t+1}^M + p_G\left(G_{t+1} - \hat{G}_{t+1}^M\right) + p_N N_t,$$

  which is different from the general specification in (6) in that the term $p_\theta\left(\theta_{t+1} - \hat{\theta}_{t+1}^M\right)$ does not appear.

- **Mixed equilibrium**. It is also possible to have a mixed equilibrium with a fraction of the investors acquiring information about the government noise and the others about the asset fundamental. In such a mixed equilibrium, the general price function specified in (6) prevails.

Table I: Baseline Model Parameters

Government:	$\gamma_\sigma = 1.25,\ \gamma_\theta = 1,\ \sigma_G^2 = 2$
Asset Fundamental:	$\rho_\theta = 0.75,\ \sigma_\theta^2 = 0.01,\ \sigma_D^2 = .8$
Noise Trading:	$\rho_N = 0,\ \sigma_N^2 = 0.2$
Investors:	$\gamma = 1,\ \tau_s = 500,\ \tau_g = 500,\ R^f = 1.01$

Depending on the model parameters, all of these three types of equilibria may appear.[15] We will illustrate these equilibria in the next subsection.

3.4 Effects of Government Intervention

In this subsection we analyze effects of government intervention through a series of numerical examples. For these numerical exercises, we use a set of baseline parameter values listed in Table I. We first analyze the equilibrium when we set either γ_σ or γ_θ to be zero. Interestingly, once we set $\gamma_\sigma = 0$, the market equilibrium is invariant to the exact value of γ_θ as long as it is positive. This is because the government's optimal choice is invariant to scaling up its objective function by any positive factor. Similarly, if $\gamma_\theta = 0$, the equilibrium is also invariant to the exact value of γ_σ. Figure 3 illustrates a series of market equilibrium by varying the noise trader risk σ_N^2. Panel A depicts the conditional price variance $Var\left[P_t\left(\vartheta_{\hat{N}}\right) \mid \mathcal{F}_{t-1}^M\right]$, while Panel B depicts the conditional variance of the asset price deviation from the fundamental $Var\left[P_t\left(\vartheta_{\hat{N}}\right) - \frac{1}{R^f - \rho_\theta}\theta_{t+1} \mid \mathcal{F}_{t-1}^M\right]$. In both of these panels, the solid line corresponds to the variable of interest when $\gamma_\sigma = 0$ and $\gamma_\theta \neq 0$, while the dashed line corresponds to the variable when $\gamma_\theta = 0$ and $\gamma_\sigma \neq 0$. We also plot a short-dashed line to illustrate, as a benchmark, the Hellwig equilibrium without any government intervention (i.e., $\gamma_\sigma = \gamma_\theta = 0$).

When $\gamma_\sigma = 0$ and $\gamma_\theta \neq 0$, the government's single objective is to improve the price informativeness. In contrast, when $\gamma_\theta = 0$ and $\gamma_\sigma \neq 0$, the government's single objective is to reduce price volatility. Figure 3 shows that while these two objectives are often treated as equivalent, they lead to sharply different equilibria—a fundamental-centric equilibrium

[15] In our current setting, the government can first commit to its intervention strategy before the investors acquire their information. As a result, multiple equilibria would not arise for a given set of model parameters. If the government cannot pre-commit to its intervention strategy, multiple equilibria may arise.

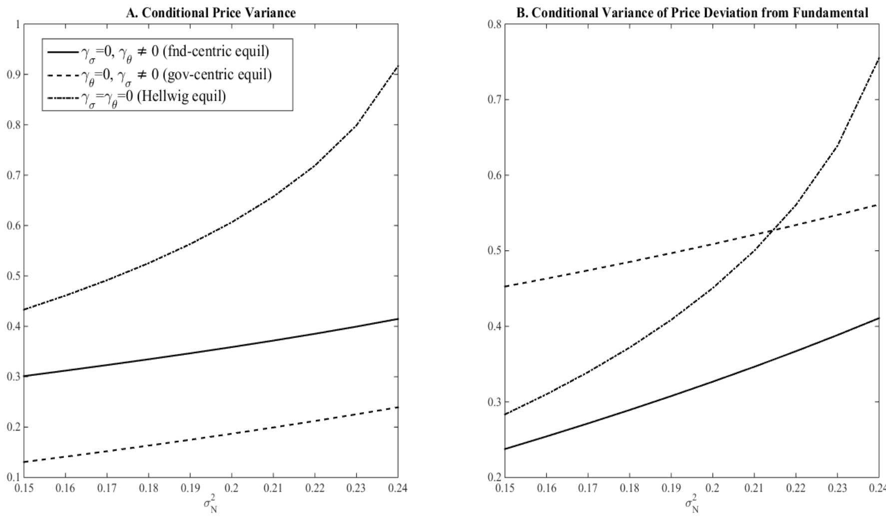

Figure 3: Equilibria across noise trader risk. This figure uses parameters listed in Table I. Panel A depicts the conditional price variance $Var\left[P_t\left(\vartheta_{\hat{N}}\right) \mid \mathcal{F}_{t-1}^M\right]$, while Panel B depicts the conditional variance of price deviation from the fundamental $Var\left[P_t\left(\vartheta_{\hat{N}}\right) - \frac{1}{R^f - \rho_\theta}\theta_{t+1} \mid \mathcal{F}_{t-1}^M\right]$. In both panels, the solid line is for the case when $\gamma_\sigma = 0$ and $\gamma_\theta \neq 0$ (i.e., the fundamental-centric equilibrium), the dahsed line for the case when $\gamma_\theta = 0$ and $\gamma_\sigma \neq 0$ (i.e., the government-centric equilibrium), and the short-dashed line for the Hellwig benchmark without any government intervention.

for the case with $\gamma_\sigma = 0$ and $\gamma_\theta \neq 0$, as shown by the solid line, and a government-centric equilibrium for the case with $\gamma_\theta = 0$ and $\gamma_\sigma \neq 0$, as shown by the dashed line. Across the different values of σ_N^2, the fundamental-centric equilibrium has uniformly higher price variance but lower variance of price deviation from fundamental. These differences reflect the investors' different information acquisition strategies in these equilibria.

In the case of $\gamma_\theta = 0$ and $\gamma_\sigma \neq 0$, a government-centric equilibrium always emerges. In this equilibrium, the government trades as much as it can to reduce the price impact of noise traders, and the price impact of its own noise G_t becomes so large relative to that of the fundamental that the investors choose to acquire private information about the government noise factor rather than the fundamental factor. Consequently, the information efficiency of the asset price is poor (i.e., the variance of price devitation from the fundamental is high) relative to the fundamental-centric equilibrium, and could become even worse than the Hellwig equilibrium without any government intervention when σ_N^2 is in the lower range of the plot in Panel B. However, the lower price variance is sufficient to fulfill the objective of the government given that $\gamma_\theta = 0$.

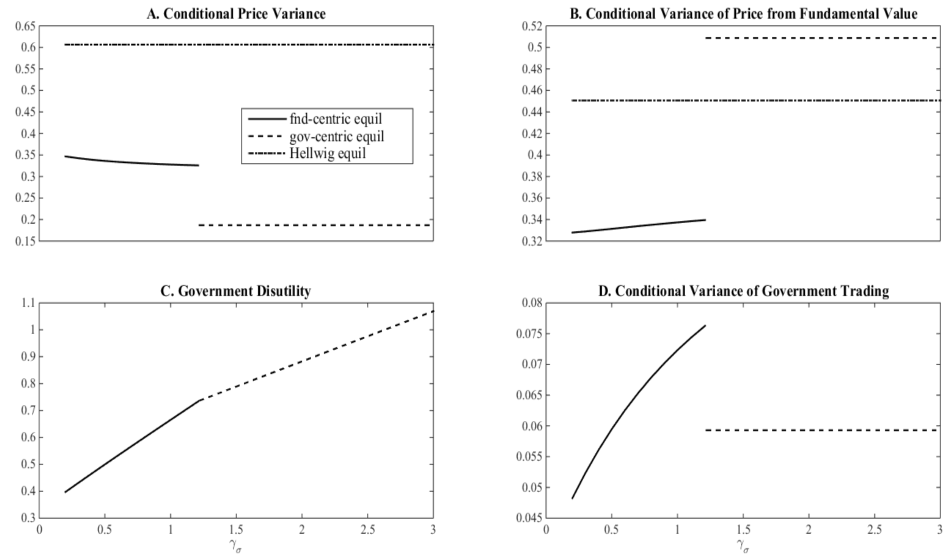

Figure 4: Equilibria across the government's objective, by setting $\gamma_\theta = 1$ and varing the value of γ_σ. The other parameters are given in Table I. Panel A depicts the conditional price variance $Var\left[P_t\left(\vartheta_{\hat{N}}\right) \mid \mathcal{F}_{t-1}^M\right]$, Panel B the conditional variance of price deviation from the fundamental $Var\left[P_t\left(\vartheta_{\hat{N}}\right) - \frac{1}{R^f - \rho_\theta}\theta_{t+1} \mid \mathcal{F}_{t-1}^M\right]$, Panel C the government's disutility, and Panel D the varaince of the government's trading. In all of these panels, the solid line corresponds to the fundamental-centric equilibrium, the dahsed line the government-centric equilibrium, and the short-dashed line the Hellwig benchmark without any government trading.

When $\gamma_\sigma = 0$ and $\gamma_\theta \neq 0$, the government's single objective is to improve the information efficiency of the asset price. Consequently, its optimal strategy is to trade against the noise traders within the limit of not making its own noise impact overly dominant so that the investors would still choose to acquire information about the fundamental. This leads to the fundamental-centric equilibrium, in which the improved information efficiency relative to that in the government-centric equilibrium comes at an expense of the greater price variance.

We now examine the equilibrium when the government has a mixed objective of improving information efficiency and reducing price variance. Specifically, Figure 4 illustrates the equilibria by setting $\gamma_\theta = 1$ and varying γ_σ from 0.2 to 3. This figure shows several interesting features. First, the fundamental-centric equilibrium emerges when γ_σ is lower than a threshold around 1.2, while a government-centric equilibrium emerges when γ_σ gets larger than the threshold. It is intuitive that the market switches from the fundamental-centric equilibrium to the government-centric equilibrium as the government assigns a greater weight to reducing price variance and thus intervenes more aggressively. Second, consistent with the illustra-

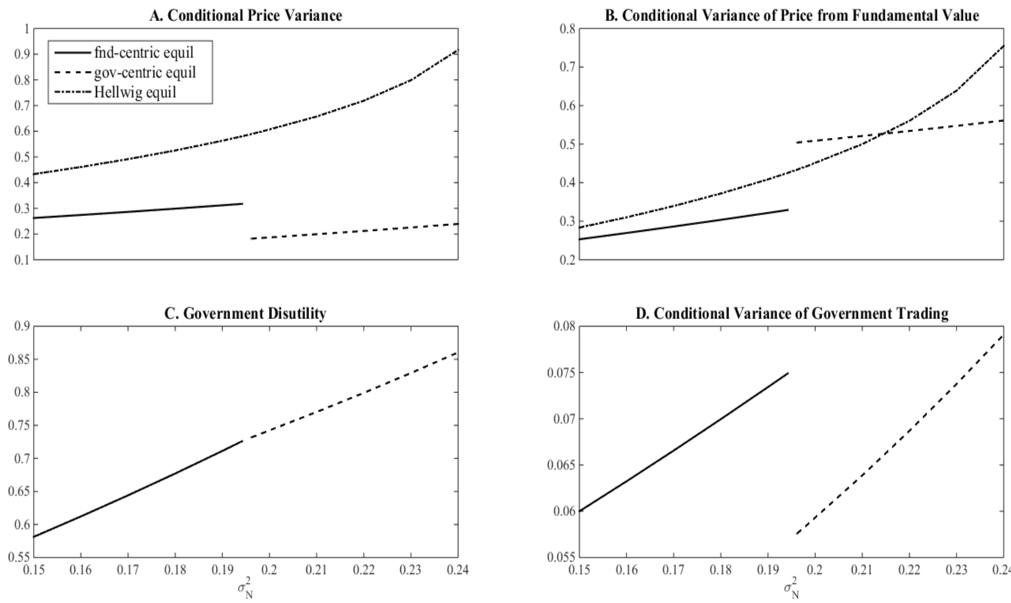

Figure 5: Equilibria across noise trader risk based on parameter values in Table I. Panel A depicts the conditional price variance $Var\left[P_t\left(\vartheta_{\hat{N}}\right) \mid \mathcal{F}^M_{t-1}\right]$, Panel B the conditional variance of price deviation from the fundamental $Var\left[P_t\left(\vartheta_{\hat{N}}\right) - \frac{1}{R^f-\rho_\theta}\theta_{t+1} \mid \mathcal{F}^M_{t-1}\right]$, Panel C the government's disutility, and Panel D the varaince of the government's asset position. In all of these panels, the solid line corresponds to the fundamental-centric equilibrium, the dahsed line the government-centric equilibrium, and the short-dashed line the Hellwig benchmark without any government trading.

tion in Figure 3, the government-centric equilibrium comes with lower price variance but worse information efficiency than the fundamental-centric equilibrium, and its information efficiency can be even worse than that in the Hellwig benchmark without any government intervention.

Third, Panel D of Figure 4 shows an interesting yet surprising observation that the government trades less in the government-centric equilibrium than in the fundamental-centric equilibrium, even though one would expect the government to trade more aggressively against noise traders in the government-centric equilibrium. This observation reflects another important dimension of the market dynamics. In the fundamental-centric equilibrium, each investor has his own private information about the asset fundamental and private information causes the investors to hold different beliefs from each other and from the government about not only the asset fundamental but also the current-period noise trading. As a result, the government has to trade against not only noise traders but also the investors. The investors' trading disseminates their private fundamental information into the asset price and improves its information efficiency, but partially offsets the government's effort to counter noise traders. In contrast, in the government-centric equilibrium, the investors' private information is about the next-period government noise, and, like the government, the investors all use the same public information to infer the current-period noise trading. Consequently, the investors tend to trade against noise traders along the same direction as the government, and thus reinforcing the effectiveness of the government's intervention in reducing volatility, although at the expense of lower informantion efficiency.

By fixing the government's objective to be a mix of reducing price variance and improving information efficiency with $\gamma_\sigma = 1.25$ and $\gamma_\theta = 1$, Figure 5 shows how the equilibrium varies with noise trader risk σ_N^2. When σ_N^2 is smaller than a threshold around 0.195, there is a fundamental-centric equilibrium. In this region, the motivation for government intervention is modest, and the government trades against noise traders to the extent not to distract the investors from acquiring information about the asset fundamental. Consequently, both the price variance and the variance of price deviation from the fundamental are lower than the Hellwig benchmark. When σ_N^2 rises above the threshold, the market switches into the government-centric equilibrium, in which the government noise factor becomes sufficiently dominant in the asset price and the investors choose to acquire information about the government noise factor rather than the asset fundamental. Consequently, there is a sharp drop in the price variance, which comes at the expense of a sharp increase in the variance of price deviation from the fundamental. Again, when the market switches from the fundamental-centric equilibrium to the government-centric equilibrium, the government trades less rather than more. As we discussed before, this is because the investors trade along with the government, which makes the government trading more effective in countering noise traders.

Figure 6 depicts the boundary between the government-centric equilibrium and the fundamental-centric equilibrium on a plane of γ_σ and σ_N^2. As the government assigns a higher weight to reducing price variance in its objective, the market shifts from the fundamental-centric equilibrium to the government-centric equilibrium at a lower value of σ_N^2.

3.5 A Time-Inconsistency Problem

[to be added.]

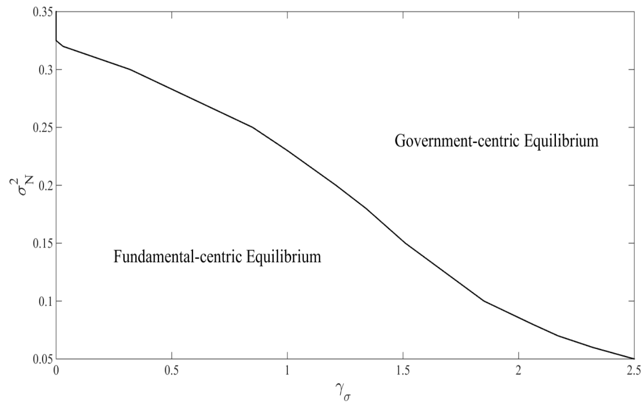

Figure 6: Equilibrium switching point based on the baseline parameter values listed in Table I.

4 Conclusion

We believe our thereotical framework captures several important features of China's financial system. First, our framework builds on a group of noise traders whose trading may cause asset price volatility to explode and even the market to break down. This feature is consistent with the joint presence of a large population of inexperienced retail investors and large asset price volatility in China's financial markets, which is often used by the Chinese government to motivate its active intervention in asset markets.

Second, intensive interventions make noise induced by the government's intervention programs an important pricing factor in asset prices. While our analysis focuses on government intervention through direct trading in asset markets, it is obviously the case that noise induced by other types of intervention programs would also induce a similar effect. Indeed, many commentators of China's financial system have pointed out the importance of government policies in driving asset market dynamics in China, even though this feature is yet to be formally examined by systematic empirical studies.

Third, as an important pricing factor, government noise, in turn, attracts speculation of short-term investors by diverting their attention away from asset fundamentals, and the investor speculation further reinforces the impact of government noise on asset prices. This feature is also consistent with a widely-held view that Chinese investors pay excessive attention to government policies, which can have a powerful impact on asset prices in the short-run, but insufficient attention to asset fundamentals, which operate over longer horizons. It would be a fruitful area of research to systematically examine this issue in the data.

Fourth, with an objective to reduce asset price volatility, intensive government intervention may move the market into a government-centric equilibrium, in which investors all focus on speculating about the noise in the government's policies while ignoring asset fundamentals. Through this channel, government intervention reduces asset price volatility at an expense of worsened information efficiency. This implication contradicts a conventional wisdom that, in trading against noise traders, the government objective of reducing price volatility is consistent with improving information efficiency. Our analysis thus cautions the government to limit the intensity of its intervention programs in order to prevent the market from shifting into a government-centric equilibrium with less volatile but also less efficient asset prices.

While our analysis is directly motivated by the intensive intervention programs pursued by the Chinese government in managing its financial system, we believe the implications of our analysis may be also relevant for other market settings with intensive government intervention. For example, many OECD countries engaged in large-scale asset purchase programs during the financial crisis and the subsequent recession. As recognized by market commentators, such government intervention programs may also susbtantially alter the dynamics of asset markets in OECD countries.

References

Allen, Frank, Stephen Morris, and Hyun Song Shin (2006), Beauty Contests and Iterated Expectations in Asset Markets, *The Review of Financial Studies* 19, 719-752.

Bacchetta, Philippe and Eric van Wincoop (2006), Can Information Heterogeneity Explain the Exchange Rate Determination Puzzle, *American Economic Review* 96, 552-576.

Bacchetta, Philippe and Eric van Wincoop (2008), Higher Order Expectations in Asset Pricing, *Journal of Money, Banking, and Credit* 40, 837-866.

Baker, Scott R., Nicholas Bloom, and Steven J. Davis (2015), Measuring Economic Policy Uncertainty, mimeo Stanford University and University of Chicago Booth School of Business.

Black, Fischer (1986), Noise, *Journal of Finance,* 41, 529-543.

Bond, Phillip and Itay Goldstein (2015), Government Intervention and Information Aggregation by Prices, *Journal of Finance,* 70, 2777-2811.

Boyarchenko, Nina, David O. Lucca, and Laura Veldkamp (2016), Taking Orders and Taking Notes: Dealer Information Sharing in Financial Markets, mimeo Federal Reserve Bank of NY and NYU Stern School of Business.

Brunnermeier, Markus (2001), *Asset Pricing Under Asymmetric Information: Bubbles, Crashes, Technical Analysis, and Herding* Oxford University Press, Oxford.

Cong, Lin William, Steven R. Grenadier, Yunzhi Hu (2017), Intervention Policy in a Dynamic Environment: Coordination and Learning, mimeo Stanford GSB and University of Chicago.

Croce, Maximiliano M., Howard Kung, Thien T. Nguyen, and Lukas Scmid (2012), Fiscal Policies and Asset Prices, *Review of Financial Studies,* 25, 2635-2672.

Croce, Maximiliano M., Thien T. Nguyen, and Lukas Scmid (2012), The Market Price of Fiscal Uncertainty, *Journal of Monetary Economics* 59, 401-416.

De Long, J. Bradford, Andrei Shleifer, Lawrence H. Summers, and Robert J. Waldmann (1990), Noise Trader Risk in Financial Markets, *Journal of Political Economy,* 98, 703-738.

Fernández-Villaverde, Jesús, Pablo Guerrón-Quintana, Keith Kuester, and Juan Rubio-Ramírez (2013), Fiscal Volatility and Economic Activity, NBER Working Paper.

Fudenberg, Drew and David K. Levine (1989), Reputation and Equilibrium Selection in Games with a Patient Player, *Econometrica,* 57, 759-778.

Gertler, Mark and Nobuhiro Kiyotaki (2013), Qe 1 vs. 2 vs. 3...: A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool, *International Journal of Central Banking*, 9, 5-53.

Goldstein, Itay and Liyan Yang (2015), Market Efficiency and Real Efficiency: The Connect and Disconnect via Feedback Effects, mimeo Upenn Wharton School and University of Toronto Rotman School of Management.

Grossman, Sanford and Joseph Stiglitz (1980), On the impossibility of informationally efficient markets, *American Economic Review* 70, 393-408.

He, Hua and Jiang Wang (1995), Differential Information and Dynamic Behavior of Stock Trading Volume, *The Review of Financial Studies* 8, 919-972.

Hellwig, Martin (1980), On the aggregation of information in competitive markets, *Journal of Economic Theory* 22, 477-498.

Kacperczyk, Marcin, Stijn van Niewerburgh, and Laura Veldkamp (2015), A Rational Theory of Mutual Funds' Attention Allocation, *Econometrica,* forthcoming.

Mei, Jianping, José Scheinkman, and Wei Xiong (2009), Speculative Trading and Stock Prices: Evidence from Chinese A-B Shares Premia, *Annals of Economics and Finance* 10, 225-255.

Pastor, Lubos and Pietro Veronesi (2012), Uncertainty about Government Policy and Stock Prices, *Journal of Finance* 67, 1219-1264.

Pastor, Lubos and Pietro Veronesi (2013), Political Uncertainty and Risk Premia, *Journal of Financial Economy* 110, 520-545.

Shleifer, Andrei and Robert Vishny (1997), The Limits of Arbitrage, *Journal of Finance,* 52, 35-55.

Sialm, Clemes (2006), Stochastic Taxation and Asset Pricing in Dynamic General Equilibrium, *Journal of Economic Dynamics and Control* 30, 511-540.

Song, Zheng, Kjetil Storesletten, and Fabrizio Zilibotti (2011), Growing Like China, *American Economic Review* 101, 202-241.

Stein, Jeremy and Adi Sundarem (2016), The Fed, the Bond Market, and Gradualism in Monetary Policy, Working paper, Harvard Univerrsity.

Uhlig, Harald (1996), A Law of Large Numbers for Large Economies, *Economic Theory* 8, 41-50.

Ulrich, Maxim (2013), How Does the Bond Market Perceive Government Interventions?, Working paper, Columbia University.

Van Niewerburgh, Stijn and Laura Veldkamp (2010), Information Acquisition and Under-Diversification, *Review of Economics Studies* 77, 779-805.

Wang, Jiang (1994), A Model of Competitive Stock Trading Volume, *Journal of Political Economy* 102, 127-168.

Xiong, Wei and Jialin Yu (2011), The Chinese Warrants Bubble, *American Economic Review* 101, 2723-2753.

Appendix A Deriving Perfect Information Equilibrium

In the benchmark equilibrium with perfect information, all investors and the government observe the asset fundamental θ_t and the noise trading N_t. Let us conjecture a linear equilibrium in which the stock price takes a linear form:

$$P_t = p_\theta \theta_t + p_N N_t.$$

Given that dividends are $D_t = \theta_t + \varepsilon_t^D$, the stock price must react to a deterministic unit shift in θ_t by the present value of dividends deriving from that shock, $\frac{1}{R^f - \rho_\theta}$, it follows that $p_\theta = \frac{1}{R^f - \rho_\theta}$. Furthermore, we can express the government's linear trading rule for its asset demand as $X_t^G = \vartheta_N N_t$. Forcing $\vartheta_N = 0$ corresponds to the case without government intervention. Since all investors are symmetrically informed, they will have identical demand for the asset $x_t^i = x_t^S$, which, along with the government's trading, accommodates the trading of noise traders. Given that investors have myopic CARA preferences, and that dividends and prices are linear in θ_t and N_t, and therefore also normally distributed, it follows that their optimal trading policy is to have a mean-variance demand for the risky asset:

$$X_t^S = \frac{1}{\gamma} \frac{E\left[D_{t+1} + \Delta P_{t+1} \mid \mathcal{F}_t\right]}{Var\left[D_{t+1} + \Delta P_{t+1} \mid \mathcal{F}_t\right]}.$$

Solving for the expressions $E\left[D_{t+1} + \Delta P_{t+1} \mid \mathcal{F}_t\right]$ and $Var\left[D_{t+1} + \Delta P_{t+1} \mid \mathcal{F}_t\right]$ based on the price conjecture, and imposing market clearing

$$X_t^S + \vartheta_N N_t + \vartheta_N \sigma_N G_t = N_t,$$

we arrive at several conditions that relate the coefficients ϑ_N to the price coefficients p_θ, p_g, and p_N. Importantly, these relationships will give rise to a necessary condition for the existence of an equilibrium that depends on the government's trading rule.

Finally, substituting the price conjecture and market clearing conditions into the government's optimization, we are able to simplify its optimization program. This is summarized in the following proposition.

Proposition A1 *When θ_t and N_t are observable to investors and the government, the asset price takes the linear form:*

$$P_t = \frac{1}{R^f - \rho_\theta} \theta_t + p_N N_t + p_g G_t.$$

All investors have the same demand X_t^S for the risky asset:

$$X_t^S = \frac{1}{\gamma} \frac{p_N \left(\rho_N - R^f\right) N_t - R^f p_g G_t}{\sigma_D^2 + \left(\frac{1}{R^f - \rho_\theta}\right)^2 \sigma_\theta^2 + p_N^2 \sigma_N^2 + p_g^2 \sigma_G^2},$$

and the government's demand for the risky asset $X_t^G = \vartheta^N N_t + \vartheta^N \sigma_N G_t$ *solves*

$$\begin{aligned} U^G &= \sup_{\vartheta^N} - (\gamma_\theta + \gamma_\sigma) \left(1 + \left(\frac{\rho_N - R^f}{R^f}\right)^2 \left(\frac{\vartheta_N}{1-\vartheta_N}\right)^2 \sigma_G^2\right) p_N^2 \sigma_N^2 \\ \textit{such that } \vartheta^N &= 1 - \frac{1}{\gamma} \frac{p_N \left(\rho_N - R^f\right)}{\sigma_D^2 + \left(\frac{1}{R^f - \rho_\theta}\right)^2 \sigma_\theta^2 + \left(1 + \left(\frac{\rho_N - R^f}{R^f}\right)^2 \left(\frac{\vartheta_N}{1-\vartheta_N}\right)^2 \sigma_G^2\right) p_N^2 \sigma_N^2}. \end{aligned}$$

Furthermore, the asset market breaks down whenever

$$R^f < \rho_N + 2\left(1 - \vartheta_N\right) \gamma \sqrt{\left(1 + \left(\frac{\rho_N - R^f}{R^f}\right)^2 \left(\frac{\vartheta_N}{1-\vartheta_N}\right)^2 \sigma_G^2\right) \left(\sigma_D^2 \sigma_N^2 + \left(\frac{1}{R^f - \rho_\theta}\right)^2 \sigma_\theta^2 \sigma_N^2\right)}.$$

Though we derive the model with government intervention here, in the main text we discuss the perfect information settings both with and without government intervention. In the absence of government intervention, by setting $\vartheta^N = 0$, the condition for market breakdown in Proposition A1 simplifies to

$$R^f < \rho_N + 2\gamma \sqrt{\sigma_D^2 \sigma_N^2 + \left(\frac{1}{R^f - \rho_\theta}\right)^2 \sigma_\theta^2 \sigma_N^2}.$$

In the Internet Appendix, we further establish that the linear price equilibrium is the unique equilibrium when the government follows a linear trading strategy or in the absence of any intervention. This implies that no equilibrium exists when noise trading is too volatile, and highlights the role of the government in preventing market failure.

Appendix B Deriving Equilibrium with Information Frictions and No Government Intervention

In the equilibrium, the asset price in each period aggregates the investors' private information and partially reveals the asset fundamental. Let us conjecture a linear equilibrium in which the asset price takes the linear form:

$$P_t = p_{\hat{\theta}} \hat{\theta}_{t+1}^M + p_\theta \left(\theta_{t+1} - \hat{\theta}_{t+1}^M\right) + p_N N_t.$$

In this price function, $\hat{\theta}_{t+1}^M$ represents the market belief regarding the asset fundamental based on all public information, $\theta_{t+1} - \hat{\theta}_{t+1}^M$ represents the deviation of true fundamental from the market belief, and N_t is the noise trading. As in Hellwig (1980) and Allen, Morris and Shin (2006), myopic investors trade based on their private information to accommodate noise traders and to speculate on asset returns. The asset price aggregates their private information to partially reveal θ_{t+1} and the noise trading N_t, as well as $\hat{\theta}_{t+1}^M$, which is the anchor of the fundamental value based on the market's public information.

We assume that the economy is initialized from its stationary equilibrium in which all conditional variances from learning have reached their deterministic steady state and the coefficients in prices and policies are time homogeneous. Importantly, we recognize that it must be the case that $p_{\hat{\theta}} = \frac{1}{R^f - \rho_\theta}$, since a deterministic unit shift in θ_t must raise the asset price by $\frac{1}{R^f - \rho_\theta}$, the increase in the discounted present value of the future, nonstochastic cash flows since θ_{t+1} affects prices through dividends $\{D_s\}_{s \geq t+1}$. As $\hat{\theta}^M_{t+1}$ is public information, the information content of P_t is equivalent to

$$\eta^H_t = P_t - (p_{\hat{\theta}} - p_\theta)\,\hat{\theta}^M_{t+1} = p_\theta \theta_{t+1} + p_N N_t.$$

The following proposition characterizes the market equilibrium.

Proposition A2 *When the government is absent from the asset market, the linear equilibrium asset price takes the form:*

$$P_t = \frac{1}{R^f - \rho_\theta} \hat{\theta}^M_{t+1} + p_\theta \left(\theta_{t+1} - \hat{\theta}^M_{t+1} \right) + p_N N_t,$$

where p_θ and p_N satisfy

$$\begin{aligned} \varphi' \begin{bmatrix} 1 \\ p_\theta (\rho_\theta - \rho_N) \end{bmatrix} \frac{\Sigma^{M,\theta\theta}}{\Sigma^{M,\theta\theta} + \tau_s^{-1}} + \left(\rho_N - R^f \right) p_\theta &= 0, \\ \frac{\left(\rho_N - R^f \right) p_N}{\gamma \varphi' \Omega(i) \varphi} &= 1. \end{aligned}$$

Futhermore, the common belief held by the market participants about θ_{t+1}, $\hat{\theta}^M_{t+1}$, evolves according to

$$\hat{\theta}^M_{t+1} = \rho_\theta \hat{\theta}^M_t + \begin{bmatrix} 1 \\ 0 \end{bmatrix}' \mathbf{k}^M \begin{bmatrix} D_t - \hat{\theta}^M_t \\ \eta^H_t - p_\theta \rho_\theta \hat{\theta}^M_t - p_N \rho_N \hat{N}^M_{t-1} \end{bmatrix},$$

with the conditional variance $\Sigma^{M,\theta\theta}$ solving a Ricatti equation. The private belief of investor i, $\hat{\theta}^i_{t+1}$, is related to the market belief by

$$\hat{\theta}^i_{t+1} = \hat{\theta}^M_{t+1} + \frac{\Sigma^{M,\theta\theta}}{\Sigma^{M,\theta\theta} + \tau_s^{-1}} \left(s^i_t - \hat{\theta}^M_{t+1} \right).$$

Finally, the demand for the risky asset of investor i is given by

$$X^i_t = \frac{\left(1 + p_\theta \left(\rho_\theta - R^f \right) + \begin{bmatrix} p_{\hat{\theta}} - p_\theta \\ 0 \end{bmatrix}' \mathbf{k}^M \begin{bmatrix} 1 \\ p_\theta (\rho_\theta - \rho_N) \end{bmatrix} \right) \left(\hat{\theta}^i_{t+1} - \hat{\theta}^M_{t+1} \right) + p_N \left(\rho_N - R^f \right) \hat{N}^i_t}{\gamma \varphi' \Omega(i) \varphi},$$

with $\mathbf{k}^M$, φ, and $\Omega(i)$ given in the online Appendix.

We can make an additional insight about the equilibrium with asymmetric information without government intervention. By rearranging the condition for p_θ in Proposition A2, we see that

$$p_\theta = \frac{1}{1+\frac{R^f-\rho_N}{R^f-\rho_\theta}\frac{\tau_s^{-1}}{\Sigma^{M,\theta\theta}}}\frac{1}{R^f-\rho_\theta} \le \frac{1}{R^f-\rho_\theta},$$

which is its value under perfect information. Consequently, in the presence of information frictions, the asset fundamental θ_t is less reflected in the asset price than justified by its fundamental value. This can be motivation for government intervention to promote not only price stability, but also the information efficiency of the asset price.

Appendix C Deriving Equilibrium with Information Frictions and Government Intervention

In this Appendix, we build up the solution to the equilibrium with information frictions and government intervention in several steps. We assume that the economy is initialized from its stationary equilibrium in which all conditional variances from learning have reached their deterministic steady state and the coefficients in prices and policies are time homogeneous.

We begin, as in the main text, by conjecturing a linear equilibrium price function:

$$P_t = p_{\hat{\theta}}\hat{\theta}^M_{t+1} + p_{\hat{G}}\hat{G}^M_{t+1} + p_\theta\left(\theta_{t+1}-\hat{\theta}^M_{t+1}\right) + p_G\left(G_{t+1}-\hat{G}^M_{t+1}\right) + p_g G_t + p_N N_t.$$

Importantly, we recognize that it must be the case that $p_{\hat{\theta}} = \frac{1}{R^f-\rho_\theta}$, since a deterministic unit shift in θ_t must raise the asset price by $\frac{1}{R^f-\rho_\theta}$, the increase in the discounted present value of the future cash flows.

We now construct the equilibrium in several steps. We first solve for the learning processes of the government and investors, which begin with an intermediate step of deriving the beliefs from the perspective of the market that has access to only public information. Given the market's beliefs, which we can define recursively with the Kalman Filter, we can construct the conditional posterior beliefs of the government, and the posterior beliefs of each investor by applying Bayes' Rule to the market's beliefs given the private signal of each investor. We then solve for the optimal trading and information acquisition policies of the investors. Imposing market clearing, we can then express the government's objective in terms of the equilibrium objects we derive from learning.

C.1. Equilibrium Beliefs

In this subsection, we characterize the learning processes of the government and the investors. As we will see, it will be convenient to first derive the market's posterior beliefs about θ_{t+1}, N_t, and G_{t+1}, respectively, which will be Gaussian with conditional expectation $\left(\hat{\theta}^M_t, \hat{N}^M_t, \hat{G}^M_{t+1}\right) = E\left[(\theta_t, N_t, G_{t+1}) \mid \mathcal{F}^M_t\right]$ and conditional variance $\Sigma^M_t = Var\left[\begin{bmatrix}\theta_t \\ N_t \\ G_{t+1}\end{bmatrix} \mid \mathcal{F}^M_t\right]$.

Importantly, the market faces strategic uncertainty over the government's action due to the noise in the government's trading. As such, one must form expectations about this noise both for extracting information from prices and for understanding price dynamics for portfolio choice.

To solve for the market beliefs, we first construct the innovation process η_t^M for the asset price from the perspective of the market

$$\begin{aligned}\eta_t^M &= P_t - \left(p_{\hat{\theta}} - p_\theta\right)\hat{\theta}_{t+1}^M - \left(p_{\hat{G}} - p_G\right)\hat{G}_{t+1}^M - p_g G_t \\ &= p_\theta \theta_{t+1} + p_G G_{t+1} + p_N N_t.\end{aligned}$$

Given that the investors and the government do not observe G_{t+1} the next-period government noise, they cannot extract it from the asset price and must account for it in their learning.

Importantly, the asset price P_t and the innovation process η_t^M contain the same information, so that $\mathcal{F}_t^M = \sigma\left(\left\{D_s, \eta_s^M\right\}_{s\le t}\right)$. Since the market's posterior about θ_{t+1} will be Gaussian, we need only specify the laws of motion for the conditional expectation $\left(\hat{\theta}_{t+1}^M, \hat{N}_t^M, \hat{G}_{t+1}^M\right)$ and the conditional variance Σ_t^M. As is standard with a Gaussian information structure, these estimates are governed by the Kalman Filter. As a result of learning from prices, the beliefs of the market about θ_{t+1}, N_t, and G_{t+1} will be correlated ex-post after observing the asset price. We summarize this result in the following proposition.

Proposition A3 *Given the normal prior* $(\theta_0, N_0) \sim \mathcal{N}\left(\left(\bar{\theta}, \bar{N}\right), \Sigma_0\right)$ *and* $G_0 \sim \mathcal{N}\left(0, \sigma_G^2\right)$, *the posterior market beliefs are Gaussian* $(\theta_{t+1}, N_t, G_{t+1}) \mid \mathcal{F}_t^M \sim \mathcal{N}\left(\left(\hat{\theta}_{t+1}^M, \hat{N}_t^M, \hat{G}_{t+1}^M\right), \Sigma_{t+1}^M\right)$, *where the filtered estimates* $\left(\hat{\theta}_t^M, \hat{N}_t^M, \hat{G}_{t+1}^M\right)$ *follow the stochastic difference equations*

$$\begin{bmatrix} \hat{\theta}_{t+1}^M \\ \hat{N}_t^M \\ \hat{G}_{t+1}^M \end{bmatrix} = \begin{bmatrix} \rho_\theta \hat{\theta}_t^M \\ \rho_N \hat{N}_{t-1}^M \\ 0 \end{bmatrix} + \mathbf{K}_t^M \begin{bmatrix} D_t - \hat{\theta}_t^M \\ \eta_t^M - p_\theta \rho_\theta \hat{\theta}_t^M - p_N \rho_N \hat{N}_{t-1}^M \end{bmatrix},$$

and the conditional variance $\Sigma_t^M = Var\left[\left[\begin{array}{c} \theta_t - \hat{\theta}_t^M \\ N_t - \hat{N}_t^M \\ G_{t+1} - \hat{G}_{t+1}^M \end{array}\right] \middle| \mathcal{F}_t^M\right]$ *follows a deterministic induction equation. The market's posterior expectations and variance of* θ_{t+1}, N_t, *and* G_{t+1} *are related through*

$$p_\theta \theta_{t+1} + p_G G_{t+1} + p_N N_t = p_\theta \hat{\theta}_{t+1}^M + p_G \hat{G}_{t+1}^M + p_N \hat{N}_t^M.$$

Importantly, when the market tries to extract information from the price, market participants realize that the price innovations η_t^M contain the government noise G_{t+1}. As such, they must take into account the information content in the government noise when learning from the price, and must form expectations about G_{t+1}. Through this channel, the path dependence of the government noise feeds into the market's beliefs and the market has incentives to forecast the future government noise.

Since investors learn through Bayesian updating, we can update their beliefs sequentially by beginning with the market beliefs, based on the coarser information set $\mathcal{F}_t^M$, and then updating the market beliefs with the private signals of investor i, $\left(s_t^i, g_t^i\right)$. Given that the market posterior beliefs and investor private signals are Gaussian, this second updating process again takes the form of a linear updating rule. We summarize these steps in the following proposition.

Proposition A4 *Given the market beliefs, the conditional beliefs of investor i are also Gaussian $(\theta_t, N_t, G_{t+1}) \mid \mathcal{F}_t^i \sim \mathcal{N}\left(\left(\hat{\theta}_t^i, \hat{N}_t^i, \hat{G}_{t+1|t}^i\right), \Sigma_t^s(i)\right)$, where*

$$\begin{bmatrix} \hat{\theta}_{t+1}^i \\ \hat{N}_t^i \\ \hat{G}_{t+1}^i \end{bmatrix} = \begin{bmatrix} \hat{\theta}_{t+1}^M \\ \hat{N}_t^M \\ \hat{G}_{t+1}^M \end{bmatrix} + \Gamma_t' \begin{bmatrix} s_t^i - \hat{\theta}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix},$$

and $\Sigma_t^s(i)$ is related to Σ_t^M through a linear updating rule.

Since the government does not observe any private information, its conditional posterior beliefs align with those of the market. In what follows, we focus on the covariance-stationary limit of the Kalman Filter, after initial conditions have died out and the conditional variances of beliefs have converged to their deterministic, steady state. The following corollary establishes such a steady state exists.

Corollary 1 *There exists a covariance-stationary stationary equilibrium, in which the conditional variance of the market beliefs has a deterministic steady state. Given this steady state, the beliefs of investors are also covariance-stationary.*

Having characterized learning by investors and the government in this economy, we now turn to the optimal policies of investors.

C.2. Investment and Information Acquisition Policies

We now examine the optimal policies of an individual investor i at date t who takes the intervention policy of the government as given. Given the CARA-normal structure of each investor's problem at date t, a separation principle applies and we can separate the investor's learning process about $(\theta_{t+1}, N_t, G_{t+1})$ from his optimal trading policy. To derive the optimal investment policy, it is convenient to decompose the excess asset return as

$$R_{t+1} = E\left[R_{t+1} \mid \mathcal{F}_t^M\right] + \phi' \varepsilon_{t+1}^M = \varsigma \Psi_t^M + \phi' \varepsilon_{t+1}^M,$$

where

$$\varepsilon_{t+1}^M = \begin{bmatrix} D_{t+1} - \hat{\theta}_{t+1}^M \\ \eta_{t+1}^M - p_\theta \rho_\theta \hat{\theta}_{t+1}^M - p_N \rho_N \hat{N}_t^M - p_g \hat{G}_{t+1}^M \end{bmatrix},$$

and $\varepsilon_{t+1}^M \sim N\left(0_{2\times 1}, \Omega^M\right)$ from Proposition A3. We can then decompose the excess return based on the information set of the investor:

$$R_{t+1} = E\left[R_{t+1} \mid \mathcal{F}_t^i\right] + \phi' \varepsilon_{t+1}^S,$$

where we can update $E\left[R_{t+1} \mid \mathcal{F}_t^i\right]$ from $E\left[R_{t+1} \mid \mathcal{F}_t^M\right]$ by the Bayes' Rule according to

$$\begin{aligned}
& E\left[R_{t+1} \mid \mathcal{F}_t^M, a_t^i s_t^i + \left(1-a_t^i\right) g_t^i\right] \\
= \; & E\left[R_{t+1} \mid \mathcal{F}_t^M\right] + Cov\left[R_{t+1}, \begin{bmatrix} s_t^i - E\left[s_t^i \mid \mathcal{F}_t^M\right] \\ g_t^i - E\left[g_t^i \mid \mathcal{F}_t^M\right] \end{bmatrix}' \mid \mathcal{F}_t^M\right] \\
& \cdot Var\left[\begin{bmatrix} s_t^i - \hat{\theta}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix} \mid \mathcal{F}_t^M\right]^{-1} \begin{bmatrix} s_t^i - E\left[s_t^i \mid \mathcal{F}_t^M\right] \\ g_t^i - E\left[g_t^i \mid \mathcal{F}_t^M\right] \end{bmatrix} \\
= \; & \varsigma \Psi_t^M + \frac{\phi'\omega \begin{bmatrix} \Sigma^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,\theta G_1} \\ -\Sigma^{M,\theta G_1} & \Sigma^{M,\theta\theta} + \left(a^i\tau_s\right)^{-1} \end{bmatrix}}{\left(\Sigma^{M,\theta\theta} + (a\tau_s)^{-1}\right)\left(\Sigma^{M,G_1G_1} + \left[(1-a)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,\theta G_1}\right)^2} \begin{bmatrix} s_t^i - \hat{\theta}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix}.
\end{aligned}$$

This expression shows that the investor's private information in either s_t^i or g_t^i can help it better predict the excess asset return relative to the market information. Since investors are myopic, their optimal trading strategy is to acquire a mean-variance efficient portfolio based on their beliefs. This is summarized in the following proposition.

Proposition A5 *Given the state vector* $\Psi_t = \left[\hat{\theta}_{t+1}^M, \hat{N}_t^M, G_t, \hat{G}_{t+1}^M\right]$ *and investor i's signals* s_t^i *and* g_t^i, *investor i's optimal investment policy* X_t^i *takes the following form:*

$$X_t^i = \frac{1}{\gamma} \frac{\varsigma \Psi_t^M + \frac{\phi'\omega \begin{bmatrix} \Sigma^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,\theta G_1} \\ -\Sigma^{M,\theta G_1} & \Sigma^{M,\theta\theta} + \left(a^i\tau_s\right)^{-1} \end{bmatrix} \begin{bmatrix} s_t^i - \hat{\theta}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix}}{\left(\Sigma^{M,\theta\theta} + (a\tau_s)^{-1}\right)\left(\Sigma^{M,G_1G_1} + \left[(1-a)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,\theta G_1}\right)^2}}{\phi'\Omega^M\phi - \frac{\phi'\omega \begin{bmatrix} \Sigma^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,\theta G_1} \\ -\Sigma^{M,\theta G_1} & \Sigma^{M,\theta\theta} + \left(a^i\tau_s\right)^{-1} \end{bmatrix} \omega'\phi}{\left(\Sigma^{M,\theta\theta} + (a\tau_s)^{-1}\right)\left(\Sigma^{M,G_1G_1} + \left[(1-a)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,\theta G_1}\right)^2}},$$

wtih the coefficients ς, ϕ, *and* ω *given in the online Appendix.*

This proposition shows that both signals s_t^i and g_t^i help the investor in predicting the asset return over the public information. This is because by using s_t^i and g_t^i to form better predictions of θ_{t+1} and G_{t+1} in the current period, the investor can better predict θ_{t+1} and G_{t+1}, which determine the asset return in the subsequent period. The investor needs to choose acquiring either s_t^i or g_t^i based on the ex ante market information:

$$\begin{aligned}
E\left[U_t^i \mid \mathcal{F}_{t-1}^M\right] &= -\sup_{a_t^i \in \{0,1\}} E\left\{E\left[\exp\left(-\gamma R^f \bar{W} - \frac{1}{2}\frac{E\left[R_{t+1} \mid \mathcal{F}_t^i\right]^2}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}\right) \middle| \mathcal{F}_t^M\right] \middle| \mathcal{F}_{t-1}^M\right\} \\
&= -\sup_{a_t^i \in \{0,1\}} \sqrt{\frac{\phi'\left(\Omega^M - M(a)\right)\phi}{\phi'\Omega^M\phi}} E\left\{\exp\left(-\gamma R^f \bar{W} - \frac{\frac{1}{2}\left(\varsigma\Psi_t^M\right)^2}{\phi'\Omega^M\phi}\right) \middle| \mathcal{F}_{t-1}^M\right\},
\end{aligned}$$

where

$$M(a) = \frac{\omega \begin{bmatrix} \Sigma^{M,G_1G_1} + \left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,\theta G_1} \\ -\Sigma^{M,\theta G_1} & \Sigma^{M,\theta\theta} + \left(a^i\tau_s\right)^{-1} \end{bmatrix} \omega'}{\left(\Sigma^{M,\theta\theta} + (a\tau_s)^{-1}\right)\left(\Sigma^{M,G_1G_1} + \left[(1-a)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,\theta G_1}\right)^2}.$$

This is the expected utility of investor i based on the public information from the previous period. Importantly, we recognize that the investor's information acquisition choice is independent of the expectation with respect to $\mathcal{F}_{t-1}^{M}$. Intuitively, second moments are deterministic in a Gaussian framework, so the investor can perfectly anticipate the level of uncertainty he will face without knowing the specific realization of the common knowledge information vector Ψ_t tomorrow. We can further derive the objective to

$$a = \arg \sup_{a \in \{0,1\}} -\log \left\{ \phi' \left[\Omega^M - M(a) \right] \phi \right\}. \tag{A1}$$

Since the optimization objective involves only variances, which are covariance-stationary, the signal choice faced by the investors is time invariant. Intuitively, given the Gaussian price distribution and exponential utility for the investors, the benefit of more precise private information lies with the reduction in uncertainty over the excess asset return.

By substituting $M(a)$ into the optimization objective, we arrive at the following result.

Proposition A6 *An investor will choose to acquire information about the asset fundamental θ_{t+1} (i.e., $a = 1$) with probability λ:*

$$\lambda = \begin{cases} 1, & \text{if } Q < 0 \\ (0,1), & \text{if } Q = 0 \\ 0, & \text{if } Q > 0, \end{cases},$$

where

$$Q = \frac{Cov\left[R_{t+1}, G_{t+1} \mid \mathcal{F}_t^M\right]^2}{\Sigma^{M,G_1G_1} + \tau_g^{-1}} - \frac{Cov\left[R_{t+1}, \theta_{t+1} \mid \mathcal{F}_t^M\right]^2}{\Sigma^{M,\theta\theta} + \tau_s^{-1}}$$

is given explicitly in the Appendix, and $\lambda \in (0,1)$ is the mixing probability when the investor is indifferent between acquiring information about the asset fundamental or the government noise.

The proposition states that the investor chooses his signal to maximize his informational advantage over the market beliefs by choosing which signal to learn from, based on the extent to which the signal reduces the conditional variance of the excess asset return. Importantly, this need not imply a preference for learning about θ_{t+1} directly, since the government's future noise G_{t+1} also contributes to the overall variance of the excess asset return. The more government's noise covaries with the unpredictable component of the asset return from the market's perspective, the more valuable is this information to the investors.[16] This is the partial equilibrium decision of each investor taking prices as given.

C.3. Market-Clearing

[16] Since a higher signal precision will reduce the conditional variance of the excess asset return but impact the expected return symmetrically because the signal is unbiased, the channel through which information acquisition affects portfolio returns is through reduction in uncertainty. Given that investors can take long or short positions without limit, the direction of the news surprise does not impact the information acquisition decision.

Given the optimal policy for each investor from Proposition A6 and the government's trading policy in (3), imposing market clearing in the asset market leads to

$$
\begin{aligned}
N &= \lambda \frac{\varsigma \Psi + \frac{\phi' \omega}{\Sigma^{M,\theta\theta} + \tau_s^{-1}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left(\theta - \hat{\theta}^M\right)}{\gamma \phi' \left(\Omega^M - \omega \begin{bmatrix} \frac{1}{\Sigma^{M,\theta\theta} + \tau_s^{-1}} & 0 \\ 0 & 0 \end{bmatrix} \omega'\right) \phi} \qquad \text{(A2)} \\
&+ (1-\lambda) \frac{\varsigma \Psi^M + \frac{\phi' \omega}{\Sigma^{M,G_1 G_1} + \tau_g^{-1}} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left(G_{+1} - \hat{G}^M_{+1}\right)}{\gamma \phi' \left(\Omega^M - \omega \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{\Sigma^{M,G_1 G_1} + \tau_g^{-1}} \end{bmatrix} \omega'\right) \phi} + \vartheta_{\hat{N}} \hat{N}_M + \sqrt{\vartheta' \left(\mathbf{K}^M \Omega^M \mathbf{K}^M\right) \vartheta} G,
\end{aligned}
$$

where $\vartheta = \begin{bmatrix} 0 & \vartheta_{\hat{N}} & 0 & 0 \end{bmatrix}'$ and we have applied the weak Law of Large Numbers that $\int_\chi s^i_t di = \theta_{t+1}$ and $\int_\chi g^i_t di = G_{t+1}$ over the arbitrary subset of the unit interval χ. In addition, we have recognized that $Var\left[\vartheta_{\hat{N}} \hat{N}^M \mid \mathcal{F}^M_{t-1}, \{a^i_t\}_i\right] = \vartheta' \mathbf{K}^M \Omega^M \mathbf{K}^M \vartheta$. Following the insights of He and Wang (1995), we can express the market clearing condition with a smaller, auxiliary state space given that expectations about θ_{t+1} and N_t are linked through the stock price p. We now recognize that

$$
\hat{N}^M_t = N_t + \frac{p_\theta}{p_N}\left(\theta_{t+1} - \hat{\theta}^M_{t+1}\right) + \frac{p_G}{p_N}\left(G_{t+1} - \hat{G}^M_{t+1}\right), \qquad \text{(A3)}
$$

from Proposition A3. This allows us to rewrite Ψ_t as the state vector $\tilde{\Psi}_t = [\hat{\theta}^M_{t+1}, \hat{G}^M_{t+1}, \theta_{t+1}, N_t, G_t, G_{t+1}]$.

Matching coefficients with our conjectured price function pins down the coefficients and confirms the linear equilibrium. Importantly, the coefficients are matched to the basis $\left\{\hat{\theta}^M_{t+1}, \theta_{t+1} - \hat{\theta}^M_{t+1}, G_{t+1} - \hat{G}^M_{t+1}, G_t, N_t\right\}$ in accordance with our conjecture on the functional form of the asset price. This yields three conditions:

$$
\begin{aligned}
0 &= -A\left(1 + p_{\hat{\theta}}\left(\rho_\theta - R^f\right)\right), \\
\vartheta_{\hat{N}} &= 1 - A p_N \left(\rho_N - R^f\right), \\
p_{\hat{G}} &= \frac{1}{R^f} p_g,
\end{aligned}
$$

where

$$
A = \frac{\lambda}{\gamma \phi' \left(\Omega^M - \omega \begin{bmatrix} \frac{1}{\Sigma^{M,\theta\theta} + \tau_s^{-1}} & 0 \\ 0 & 0 \end{bmatrix} \omega'\right) \phi} + \frac{1-\lambda}{\gamma \phi' \left(\Omega^M - \omega \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{\Sigma^{M,G_1 G_1} + \tau_g^{-1}} \end{bmatrix} \omega'\right) \phi},
$$

which pin down the relationship between the government's trading policy and the price coefficients, and

$$
\begin{aligned}
-AR^f p_g + \sqrt{\vartheta' \mathbf{K}^M \Omega^M \mathbf{K}^M \vartheta} &= 0, & \text{(A4)} \\
\frac{p_\theta}{p_N} + \lambda \frac{\frac{\phi'\omega}{\Sigma^{M,\theta\theta} + \tau_s^{-1}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}}{\gamma\phi' \left(\Omega^M - \omega \begin{bmatrix} \frac{1}{\Sigma^{M,\theta\theta}+\tau_s^{-1}} & 0 \\ 0 & 0 \end{bmatrix} \omega' \right) \phi} &= 0, & \text{(A5)} \\
\frac{p_G}{p_N} + (1-\lambda) \frac{\frac{\phi'\omega}{\Sigma^{M,G_1G_1} + \tau_g^{-1}} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{\gamma\phi' \left(\Omega^M - \omega \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{\Sigma^{M,G_1G_1}+\tau_g^{-1}} \end{bmatrix} \omega' \right) \phi} &= 0, & \text{(A6)}
\end{aligned}
$$

which pin down p_g, p_θ, and p_G and, consequently, the informativeness of the asset price given the loading on the noise trading p_N. As one can see above, since investors always take a neutral position on $\hat{\theta}_{t+1}^M$ (as it is common knowledge), the government also takes a neutral position by market-clearing. The market-clearing condition (A4) reflects that investors take an off-setting position to the noise G_t in the government's trading.

Since investors determine the extent to which their private information about θ_{t+1} and G_{t+1} is aggregated into the asset price, the government is limited in how it can impact price informativeness. This is reflected in the last two market-clearing conditions, (A5) and (A6). The second terms in these conditions are the intensities with which investors trade on their private information about θ_{t+1} and G_{t+1}, respectively. The first terms, $\frac{p_\theta}{p_N}$ and $\frac{p_G}{p_N}$, are the correlations of θ_{t+1} and G_{t+1} with the perceived level of noise-trading $\hat{N}_t^M$, as can be seen from equation (A3). Since the government trades based on $\hat{N}_t^M$, it cannot completely separate its impact on the true level of noise-trading N_t in prices from its impact on θ_{t+1} and G_{t+1}.

Given that the government internalizes its impact on prices when choosing its trading strategy $\vartheta_{\hat{N}}$, we can view its optimization problem as being over the choice of price coefficients $\{p_g, p_\theta, p_G, p_N\}$ in the price functional $P_t = p\left(\tilde{\Psi}_t\right)$, subject to the market-clearing conditions.

C.4. Optimal Government Policy

Lastly, we turn to the problem faced by the government at time t. Given that it holds the market's posterior beliefs, the government will choose a coefficient $\vartheta_{\hat{N}}$ for its intervention strategy to maximize its objective, taking as given the information acquisition decision of the investors. These results are summarized in the following proposition.

Proposition A7 *The optimal choice of $\vartheta_{\hat{N}}$ solves the steady-state optimization problem:*

$$
U^G = \sup_{\vartheta_{\hat{N}}} -\gamma_\sigma \left(\phi - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right)' \Omega^M \left(\phi - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right) - \gamma_\theta F - \psi H,
$$

where H and F are given in the online Appendix.

Given that the government internalizes its direct impact on the asset price, we can treat its optimization as being over the mapping of states $\tilde{\Psi}_t$ to prices P_t through the price functional $P_t = p\left(\tilde{\Psi}_t\right)$. While the government takes the information acquisition of investors as given, it manages investor expectations and trading policies through its impact on prices.

An equilibrium is then a fixed point for λ that satisfies Proposition A6. This completes our characterization of the linear noisy rational expectations equilibrium.

C.5. Computation of the Equilibrium

To compute equilibrium numerically, we follow the Kalman filter algorithm for the market's beliefs outlined in Proposition A3 to find the stationary equilibrium. We then solve for the portfolio choice of each investor and impose the market-clearing condition, and optimize the government's objective in choosing $\vartheta_{\hat{N}}$. Finally, we check each investor's information acquisition decision by computing the Q statistic to verify that the conjectured equilibrium is an equilibrium. We perform this optimization to search for both fundamental-centric ($\lambda = 1$) and government-centric ($\lambda = 0$) equilibria, as well as mixing equilibria ($\lambda \in (0, 1)$).