A MODEL OF SECULAR STAGNATION

Gauti B. Eggertsson and Neil R. Mehrotra

Brown University

Princeton
February, 2015
Secular Stagnation Hypothesis

I wonder if a set of older ideas … under the phrase secular stagnation are not profoundly important in understanding Japan’s experience, and may not be without relevance to America’s experience — Lawrence Summers

Original hypothesis:

▶ Alvin Hansen (1938): Suggests a permanent demand depression.
▶ Reduction in population growth and investment opportunities.
▶ Concerns about insufficient demand ended with WWII and subsequent baby boom.

Secular stagnation resurrected:

▶ Lawrence Summers (2013)
▶ Highly persistent decline in the natural rate of interest
▶ Chronically binding zero lower bound

Goal here:

▶ Formlize these ideas in a simple model
▶ Propose a OLG model in the spirit of Samuelson (1958)
Why are we so confident interest rates will rise soon?

Interest rates in the US during the Great Depression:

- Started falling in 1929 (reach zero in 1933) ......
- ...... only to increase in 1947

Started dropping in Japan in 1994:

- Remains at zero today

Why are we so confident interest rates are increasing in the next few years?

Need a framework where the answer is not baked into the cake – need a model that can account for arbitrary persistence of the recession
PREVIEW OF RESULTS
Permanently negative natural rate of interest can be triggered by:

- Permanent deleveraging shock
- Slowdown in population growth
- Increase in income inequality
- Fall in relative price of investment

Stagnation steady state

- Permanently binding zero lower bound
- Low inflation or deflation
- Permanent shortfall in output from potential – no obvious adjustment mechanism (price flexibility paradox).

Monetary and fiscal policy responses

- Raising the inflation target
- Increases in public debt
- Increases in government purchases
Outline for Presentation

1. Model
   - Endowment economy
     - deleveraging shocks, income inequality, population slowdown
     - price level determination
   - Endogenous production

2. Monetary and fiscal policy

3. Capital
   - Fall in the relative price of investment
ECONOMIC ENVIRONMENT

ENDOWMENT ECONOMY

- Time: $t = 0, 1, 2, \ldots$
- Goods: consumption good ($c$)
- Agents: 3-generations: $i \in \{y, m, o\}$
- Assets: riskless bonds ($B^i$)
- Technology: exogenous borrowing constraint $D$
Objective function:

$$\max_{y_t, m_{t+1}, o_{t+2}} U = \mathbb{E}_t \left\{ \log(C^y_t) + \beta \log(C^m_{t+1}) + \beta^2 \log(C^o_{t+2}) \right\}$$

Budget constraints:

$$C^y_t = B^y_t$$
$$C^m_{t+1} = Y^m_{t+1} - (1 + r_t)B^y_t + B^m_{t+1}$$
$$C^o_{t+2} = Y^o_{t+2} - (1 + r_{t+1})B^m_{t+1}$$
$$(1 + r_t)B^i_t \leq D_t$$
Consumption and Saving

Credit-constrained youngest generation:

\[ C_t^y = B_t^y = \frac{D_t}{1 + r_t} \]

Saving by the middle generation:

\[ \frac{1}{C_t^m} = \beta E_t \frac{1 + r_t}{C_{t+1}^o} \]

Spending by the old:

\[ C_t^o = Y_t^o - (1 + r_{t-1})B_{t-1}^m \]
Determination of the Real Interest Rate

Asset market equilibrium:

\[ N_t B^y_t = -N_{t-1} B^m_t \]
\[ (1 + g_t) B^y_t = -B^m_t \]

Demand and supply of loans:

\[ L^d_t = \frac{1 + g_t}{1 + r_t} D_t \]
\[ L^s_t = \frac{\beta}{1 + \beta} (Y_t^m - D_{t-1}) - \frac{1}{1 + \beta} \frac{Y_{t+1}^o}{1 + r_t} \]
Determination of the Real Interest Rate

Expression for the real interest rate (perfect foresight):

$$1 + r_t = \frac{1 + \beta (1 + g_t)D_t}{\beta Y^m_t - D_{t-1}} + \frac{1}{\beta Y^m_t - D_{t-1}} Y^o_{t+1}$$

Determinants of the real interest rate:

- Tighter collateral constraint reduces the real interest rate
- Lower rate of population growth reduces the real interest rate
- Higher middle age income reduces real interest rate
- Higher old income increases real interest rate
Effect of a Deleveraging Shock

Impact effect:
- Collateral constraint tightens from $D_h$ to $D_l$
- Reduction in the loan demand and fall in real rate
- Akin to Eggertsson and Krugman (2012)

Delayed effect:
- Next period, a shift out in loan supply
- Further reduction in real interest rate
- Novel effect from Eggertsson and Krugman (2012)
- Potentially powerful propagation mechanism
Effect of a Deleveraging Shock

![Graph showing the effect of a deleveraging shock on loan supply and demand. The graph plots the gross real interest rate against loans, with loan supply and demand curves indicated. Points A, B, C, and D represent different scenarios under the deleveraging shock.]
INCOME INEQUALITY

Does inequality affect the real interest rate?

▶ Our result due to generational inequality that triggers borrowing and lending
▶ What about inequality within a given cohort?
  - Irrelevant if output of each individual same over time
  - Easy to come up with examples where it matter

Generalization of endowment process:

▶ High-type households with high income in middle period
▶ Low-type households with low income in middle period
▶ Both types receive same income in last period
Income Inequality and Real Interest Rate

Credit constrained middle income:

- Fraction $\eta_s$ of middle income households are credit constrained
- True for low enough income in middle generation and high enough income in retirement
- Fraction $1 - \eta_s$ lend to both young and constrained middle-generation households

Expression for the real interest rate:

$$1 + r_t = \frac{1 + \beta}{\beta} \frac{(1 + g_t + \eta_s) D_t}{(1 - \eta_s) \left( Y_{t+1}^{m,h} - D_{t-1} \right)} + \frac{1}{\beta (1 - \eta_s)} \frac{Y_{t+1}^o}{\left( Y_t^{m,h} - D_{t-1} \right)}$$
PRICE LEVEL DETERMINATION

Euler equation for nominal bonds:

\[
\frac{1}{C_t^m} = \beta \mathbb{E}_t \frac{1}{C_{t+1}^o} (1 + i_t) \frac{P_t}{P_{t+1}}
\]

\[i_t \geq 0\]

Bound on steady state inflation:

\[\bar{\Pi} \geq \frac{1}{1 + r}\]

▶ If steady state real rate is negative, steady state inflation must be positive
▶ No equilibrium with stable inflation
▶ But what happens when prices are NOT flexible and the central bank does not tolerate inflation?
▶ Then the central bank’s refusal to tolerate high enough inflation will show up as a permanent recession.
ENDOGENOUS PRODUCTION - AGGREGATE SUPPLY - FULL EMPLOYMENT

Output and labor demand:

- Labor only factor of production (capital coming up)
- Firms are perfectly competitive

\[
Y_t = L_t^\alpha \\
\frac{W_t}{P_t} = \alpha L_t^{\alpha - 1}
\]

Labor supply:

- Middle-generation households supply a constant level of labor $\bar{L}$
- Implies a constant market clearing real wage $\bar{W} = \alpha \bar{L}^{\alpha - 1}$
- Implies a constant full-employment level of output: $Y_{fe} = \bar{L}^\alpha$
**Downward Nominal Wage Rigidity**

Partial wage adjustment:

\[ W_t = \max \left\{ \tilde{W}_t, P_t \alpha \bar{L}^{\alpha - 1} \right\} \]

where \( \tilde{W}_t = \gamma W_{t-1} + (1 - \gamma) P_t \alpha \bar{L}^{\alpha - 1} \)

Wage rigidity and unemployment:

- \( \tilde{W}_t \) is a wage norm
- If real wages exceed market clearing level, employment is rationed
- Unemployment: \( U_t = \bar{L} - L_t \)
- Similar assumption in Kocherlakota (2013) and Schmitt-Grohe and Uribe (2013)
For positive steady state inflation:

\[ Y = Y_{fe} = \bar{L}^{\alpha} \]

For steady state deflation:

\[ \frac{Y}{Y_{fe}} = \left( \frac{1 - \gamma}{1 - \Pi} \right)^{\frac{\alpha}{1 - \alpha}} \]

- Upward sloping relationship between inflation and output
- Vertical line at full-employment
Aggregate Supply Relation

![Graph showing the relation between aggregate supply and output.](image-url)

- **Gross Inflation Rate**
- **Output**

### Graph Details:
- The graph illustrates the relationship between aggregate supply and output.
- The x-axis represents output, ranging from 0.80 to 1.10.
- The y-axis represents gross inflation rate, ranging from 0.80 to 1.20.

### Key Points:
- The aggregate supply curve starts at a lower output value and rises as output increases, indicating a positive relationship.
- The curve is linear, showing a steady increase in gross inflation rate with increasing output.

---

**Note:** The graph is a visual representation to aid understanding the relationship between aggregate supply and output.
Derivation of Aggregate Demand

Monetary policy rule:

\[ 1 + i_t = \max \left( 1, (1 + i^*) \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \right) \]

Above binding ZLB:

\[ \frac{1 + i^*}{\Pi_{t+1}} \left( \frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} = \frac{1 + \beta (1 + g_t)D_t}{\beta Y_t - D_{t-1}} \]

Binding ZLB:

\[ \frac{1}{\Pi_{t+1}} = \frac{1 + \beta (1 + g_t)D_t}{\beta Y_t - D_{t-1}} \]
**Full Employment Steady State**

![Graph showing Full Employment Steady State](graph.png)

- **Gross Inflation Rate** vs. **Output**
- **Aggregate Supply**
- **Aggregate Demand**
- **FE Steady State**

Parameter Values

---

Page 21 / 35
Effect of a Collateral Shock

The diagram illustrates the impact of a collateral shock on the aggregate supply curve, labeled as $AD_2$. The graph shows the relationship between output and gross inflation rate, with the aggregate supply curve intersecting with the deflation steady state. The diagram highlights the shift from an initial state to a new equilibrium, demonstrating the economy's response to the shock.
Properties of the Stagnation Steady State

Long slump:
- Binding zero lower bound so long as natural rate is negative
- Deflation raises real wages above market-clearing level
- Output persistently below full-employment level

Existence and stability:
- Secular stagnation steady state exists so long as $\gamma > 0$
- If $\Pi^* = 1$, secular stagnation steady state is unique and determinate
- Contrast to deflation steady state emphasized in Benhabib, Schmitt-Grohe and Uribe (2001)
- Can do comparative statistics!
PARADOX OF FLEXIBILITY

- No obvious adjustment mechanisms to full employment
- As wages get more flexible output drops more
PARADOX OF TOIL

- Say’s Law inverted: Destroying Aggregate supply creates Aggregate Demand
- Hysteresis/Reduction in labor force participation stabilizing (reduces deflationary pressures)

[Diagram showing the relationship between Gross Inflation Rate and Output, with lines representing different states: AD2 with Deflation Steady State, AS1 with High Productivity Steady State, and AS2 with different states at various output levels.]
MONETARY POLICY RESPONSES

Forward guidance:

▶ Extended commitment to keep nominal rates low?
▶ Ineffective if households/firms expect rates to remain low indefinitely

Raising the inflation target:

▶ For sufficiently high inflation target, full employment steady state exists.
▶ Timidity trap (Krugman (2014))
▶ Multiple determinate steady states (secular stagnation and reflation)
▶ Monetary policy not as powerful as in earlier models because no way to exclude secular stagnation.
RAISING THE INFLATION TARGET

Gross Inflation Rate

Output

AD₁

AD₂

AD₃

Aggregate Supply

Deflation Steady State

Full Employment Steady State

AD₁

AD₂

AD₃
Fiscal policy and the real interest rate:

\[
L^d_t = \frac{1 + g_t}{1 + r_t} D_t + B^g_t
\]

\[
L^s_t = \frac{\beta}{1 + \beta} (Y^m_t - D_{t-1} - T^m_t) - \frac{1}{1 + \beta} \frac{Y^o_{t+1} - T^o_{t+1}}{1 + r_t}
\]

Government budget constraint:

\[
B^g_t + T^y_t (1 + g_t) + T^m_t + \frac{1}{1 + g_{t-1}} T^o_t = G_t + \frac{1 + r_t}{1 + g_{t-1}} B^g_{t-1}
\]

Fiscal instruments:

\[G_t, B^g_t, T^y_t, T^m_t, T^o_t\]
TEMPORARY INCREASE IN PUBLIC DEBT

Under constant population and set $G_t = T_t^y = B_{t-1}^g = 0$:

$$T_t^m = -B_t^g$$

$$T_{t+1}^o = (1 + r_t) B_t^g$$

Implications for natural rate:

- Loan demand and loan supply effects cancel out
- Temporary increases in public debt ineffective in raising real rate
- Temporary monetary expansion equivalent to temporary expansion in public debt at the zero lower bound
- Effect of an increase in public debt depends on beliefs about future fiscal policy
PERMANENT INCREASE IN PUBLIC DEBT (OR INVERSELY, AUSTERITY MEASURES)

Consider steady state following fiscal rule:

\[ T^o = \beta (1 + r) T^m \]
\[ L^d = \frac{1 + g}{1 + r} D + B^g \]
\[ L^s = \frac{\beta}{1 + \beta} (Y^m - D) - \frac{1}{1 + \beta} \frac{Y^o}{1 + r} \]

Implications for natural rate:

- Changes in taxation have no effects on loan supply
- Permanent rise in public debt always raises the real rate
- Equivalent to helicopter drop at the zero lower bound
- Public debt circumvents the tightening credit friction (Woodford (1990))
EXPANSIONARY FISCAL POLICY

The graph illustrates the relationship between output and gross inflation rate under expansionary fiscal policy. The lines AD2 and AD3 represent different levels of aggregate demand, each associated with a steady state of the economy. The point where the lines intersect the vertical axis at the gross inflation rate of 0.80 represents the deflation steady state. The aggregate supply line indicates the full employment steady state. The graph helps visualize the impact of expansionary fiscal policy on the economy's output and inflation rates.
GOVERNMENT PURCHASES MULTIPLIER

Slope of the AD and AS curves:

\[ \psi = \frac{1 + \beta}{\beta} (1 + g) D \]

\[ \kappa = \frac{1 - \alpha}{\alpha} \frac{1 - \gamma}{\gamma} \]

Purchases multiplier at the zero lower bound:

<table>
<thead>
<tr>
<th>Financing</th>
<th>Multiplier</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Increase in public debt</td>
<td>( \frac{1 + \beta}{\beta} \frac{1}{1 - \kappa \psi} )</td>
<td>&gt; 2</td>
</tr>
<tr>
<td>Tax on young generation</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Tax on middle generation</td>
<td>( \frac{1}{1 - \kappa \psi} )</td>
<td>&gt; 1</td>
</tr>
<tr>
<td>Tax on old generation</td>
<td>(- \frac{1 + g}{\beta} \frac{1}{1 - \kappa \psi})</td>
<td>&lt; 0</td>
</tr>
</tbody>
</table>
Rental rate and real interest rate:

\[ r_t^k = p_t^k - p_{t+1}^k \frac{1 - \delta}{1 + r_t} \geq 0 \]

\[ r_{ss} \geq -\delta \]

- Negative real rate now constrained by fact that rental rate must be positive

Relative price of capital goods:
- Decline in relative price of capital goods
- Need less savings to build the same capital stock
- \( \rightarrow \) downward pressure on the real interest rate.
- Global decline in price of capital goods (Karabarbounis and Neiman, 2014)
CONCLUSIONS

Policy implications:

▶ Higher inflation target needed
▶ Limits to forward guidance
▶ Role for fiscal policy
▶ Possible important implications for financial stability

Key takeaway:

▶ NOT that we will stay in a slump forever
▶ Slump of arbitrary duration
▶ OLG framework to model interest rates
GOING FORWARD

In progress:

▶ A quantitative variation of the model: stochastic transitions across age groups.
▶ Quantitatively decompose the effect of different channels on the real interest rate during the crisis.
▶ Did the bubble mask a secular stagnation?