Liquidity Regulation and Credit Booms: Theory and Evidence from China*

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Abstract

Many countries try to mitigate business cycle fluctuations by regulating the activities of their banks. We develop a theoretical framework to study the endogenous response of the banking sector and the implications for the aggregate economy. Under fairly mild conditions, we find that stricter liquidity standards can generate unintended credit booms as attempts to arbitrage the regulation change the allocation of savings across banks and the allocation of lending across markets. We then apply our framework to study recent events in China. We show that a regulatory push to increase bank liquidity and cap loan-to-deposit ratios in the late 2000s accounts for one-third of China's unprecedented credit boom and one-half of the increase in interbank interest rates over the same period. We also find strong empirical support for the cross-sectional differences between big and small banks predicted by the model.

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1 Introduction

Seeking to mitigate booms and busts, many countries regulate bank lending in relation to the quantity and composition of bank liabilities. Proponents insist that business cycle fluctuations would be more severe without these regulations but policy-makers remain wary of unintended consequences. In the words of Stanley Fischer, Vice Chairman of the U.S. Federal Reserve, a "tightening in regulation of the banking sector may push activity to other areas – and things happen." Exactly what happens, Fischer argues, is difficult to predict with existing models as there is limited theoretical work on the interactions between regulated and unregulated institutions and the economic incentives that drive them.[1] In this paper, we develop a theoretical framework which helps fill the gap between existing models and the models requested by policy-makers.

The need for such a framework is underscored by recent events in China, one of the world's largest and most rapidly growing economies. Between 2007 and 2014, the ratio of debt to GDP in China exploded from 110% to 200%. The ratio of private credit to private savings, sometimes a more conservative gauge, also rose markedly from 65% to 75% over the same period. This credit boom appears to have occurred on the heels of stricter regulation. Around 2008, Chinese regulators began enforcing an old but hitherto neglected loan-to-deposit cap which forbade banks from lending more than 75% of their deposits to non-financial borrowers. Loans to non-financials are among the least liquid financial assets on a bank's balance sheet, making loan-to-deposit caps akin to liquidity regulation. The objective of the enforcement action in China was to increase bank liquidity and keep credit growth under control, opening the door to three hypotheses. One hypothesis is that enforcement was successful: China's credit boom would have been even larger had regulators not begun enforcing the loan-to-deposit cap. Another hypothesis is that enforcement was of limited importance, perhaps because loan-to-deposit rules were not binding on the true drivers of credit growth. A third hypothesis is that enforcement failed: by enforcing the loan-to-deposit cap, regulators actually helped create the credit boom, a particularly egregious type of unintended consequence.

Of these hypotheses, the third is the most surprising and requires a coherent theory of how a policy that sounds like it will decrease credit growth – or, at worst, leave credit growth unchanged – leads instead to a credit boom. The theoretical framework we develop in this paper allows us to evaluate the viability of the third hypothesis as an equilibrium result. We find that it emerges as an equilibrium under fairly mild conditions and accounts for a

[1] Speech delivered at the 2015 Financial Stability Conference, Washington D.C., December 3, www.federalreserve.gov/newsevents/speech/fischer20151203a.htm.

quantitatively important fraction of the Chinese experience. In other words, stricter liquidity standards unintentionally helped create China's recent credit boom.

Although China is the setting for our quantitative analysis, none of the ingredients in the theory are uniquely Chinese. Our framework is one where banks engage in maturity transformation, borrowing short and lending long. This leaves them vulnerable to idiosyncratic withdrawal shocks, giving rise to an ex post interbank market where banks with insufficient liquidity can borrow from banks with surplus liquidity at an endogenously determined price. We add to this environment two features. First, banks can choose whether to manage all of their activities on a regulated balance sheet or whether to move some activity to a less regulated off-balance-sheet vehicle. We use the term "off-balance-sheet vehicle" to mean any accounting maneuvers banks can legally take to present more favorable balance sheets. Accounting rules are typically one step behind the maneuvers innovated by banks so we want to build a model which takes this feature of the real world seriously.[2] Second, the economy is served by both big and small banks, namely a big bank that internalizes the effects of its choices on other actors and a continuum of small banks that do not. A recent line of work by Corbae and D'Erasmo (2013, 2014) also distinguishes explicitly between big and small banks, albeit in a different environment with exogenous pricing on the interbank market and no possibility of off-balance-sheet vehicles.

We show that enforcing a loan-to-deposit cap in our framework can lead to a credit boom. The mechanism we uncover has two parts.

First, the loan-to-deposit cap leads small banks to engage in regulatory arbitrage. Our model predicts that the small banks have higher loan-to-deposit ratios than the large bank and are thus disproportionately affected by enforcement of the cap. In response, they find it optimal to offer a new savings instrument and manage the funds raised by this instrument in an off-balance-sheet vehicle that is not subject to the loan-to-deposit cap and that can therefore make the loans the bank cannot make without violating the cap. This constitutes shadow banking: it achieves the same type of credit intermediation as a regular bank without appearing on a regulated balance sheet. It also achieves the same type of maturity transformation as a regular bank, with long-term assets financed by short-term liabilities.

Second, the shadow banking activities of the small banks elicit a response from the big bank. Specifically, the big bank views these initiatives as a challenge to its profits. As small banks push to attract savings into off-balance-sheet instruments, they raise the interest rates on these instruments above the rates on traditional deposits and poach funding from the big

[2] Off-balance-sheet activities still exist under U.S. GAAP. To this point, it is estimated that U.S. banks would be roughly 20-25% larger under IFRS standards. This is a sizeable magnitude, especially since IFRS standards are not themselves immune.

bank. We show that the big bank's equilibrium response is both to issue its own high-return savings instruments and to modify its interbank behavior. While the first of these responses is intuitive, the second requires elaboration. Given that maturity mismatch extends to shadow banking, the interbank market remains an important source of emergency liquidity for small banks. Therefore, by making the interbank market less liquid, the big bank can compel small banks to behave less aggressively in their quest for off-balance-sheet business and thus lessen the extent to which they poach deposits. Since small banks do not internalize the effects of their choices, they are price-takers on the interbank market. The big bank, on the other hand, understands that it is large enough to shift the demand for liquidity relative to the supply, leading to sudden changes in the market-clearing interbank interest rate. With a central bank that does not automatically offset all such changes – examples abound in emerging economies and U.S. monetary history – the big bank has ex ante market power and the direct effect of its strategy is a tighter and more volatile interbank market.

The new equilibrium is characterized by a credit boom (i.e., more credit per unit of savings relative to the pre-enforcement equilibrium) as savings are reallocated across banks and lending is reallocated across markets. First, the reallocation of some savings from deposits at the big bank to the higher-return off-balance-sheet products of the small banks increases total credit because small banks and their off-balance-sheet vehicles lend more per unit of savings than the big bank. Second, the strategic interbank response of the big bank increases credit through traditional lending: rather than sitting idle on the liquidity that it intends to withhold from the interbank market, the big bank lends more to non-financial borrowers, thus contributing to the credit expansion. We call the increase in credit that culminates from these two channels a supply-side credit boom because it originates from the banks themselves. These channels would not operate if the interbank market were purely Walrasian with ex ante identical banks. They would also not operate if off-balance-sheet vehicles were precluded as small banks would mechanically switch from loans to more liquid assets in order to comply with loan-to-deposit enforcement, reducing credit as intended.

With the theoretical predictions in hand, we apply the model to China. Before calibrating to see how much of China's aggregate credit boom can be accounted for by loan-to-deposit enforcement, we present empirical evidence on the many cross-sectional predictions that our model delivers. First, small banks in China were indeed constrained by the loan-to-deposit cap whereas big banks were not. This is observed most saliently by looking at average balance data during the year rather than end-of-period ratios that are window-dressed to meet regulatory requirements. Second, small banks did indeed drive the proliferation of high-return savings instruments in China. Their issuance of these instruments – referred to more formally as wealth management products – causes, in the Granger sense, issuance by big banks. Small banks are also disproportionately more involved in off-balance-sheet issuance of wealth management products and the maturity of their products varies in a way that capitalizes on intertemporal changes in the frequency of loan-to-deposit exams. Third, the interbank market did indeed become tighter and more volatile, with big banks causing fluctuations not green-lighted by the central bank. Fourth, big banks have indeed become more aggressive in their lending to non-financials, even controlling for increases in lending plausibly attributable to other factors such as the funding of fiscal stimulus.

We conclude by establishing the quantitative importance of our theory. A calibrated version of our model shows that loan-to-deposit enforcement generates one-third of the increase in China's aggregate credit-to-savings ratio between 2007 and 2014. It also generates one-half of the increase in interbank rates over the same period. These are sizeable magnitudes. To provide a reference point, the conventional wisdom is that credit grew rapidly in China after 2007 because banks were forced to fund a RMB 4 trillion stimulus package in 2009 and 2010. However, even with a generous money multiplier calculation, we find that stimulus alone explains roughly the same fraction of the credit boom as loan-to-deposit enforcement. Using our calibrated model, we also find that shocks other than loan-to-deposit enforcement produce counterfactual correlations between interbank interest rates and spreads on wealth management products. A quantitative extension that allows for multiple, simultaneous shocks also assigns a dominant role to variation in loan-to-deposit rules and matches a broad set of moments almost perfectly.

1.1 Related Literature

Our paper is most closely related to the literature on liquidity regulation. Of particular relevance for the issues we study are Farhi, Golosov, and Tsyvinski (2007, 2009) who theoretically analyze the effect of liquidity regulation on market interest rates in a broad set of specifications and Gorton and Muir (2016) who provide a historical record of regulatory arbitrage during the U.S. National Banking Era. We contribute to this literature by showing how the effect of liquidity regulation depends on interbank market structure and by developing a theory of unintended credit booms.

Our paper also relates to a growing strand of research in economic history that highlights the importance of understanding interbank markets. Mitchener and Richardson (2016) show how a pyramid structure in U.S. interbank deposits propagated shocks during the Great Depression, Gorton and Tallman (2016) show how cooperation among members of the New York Clearinghouse helped end pre-Fed banking panics, and Frydman, Hilt, and Zhou (2015) show how a lack of cooperation with and between New York's trust companies became problematic during the Panic of 1907. Our paper relates to this literature as well as recent work by Corbae and D'Erasmo (2013, 2014) on the industrial organization of banking, although our focus is on understanding how liquidity regulation can be endogenously undermined.

Our paper also contributes to a growing literature on China's economy. See Song, Storesletten, and Zilibotti (2011), Cheremukhin, Golosov, Guriev, and Tsyvinski (2015), and the references therein for an overview. Like us, Chen, Ren, and Zha (2016) and Acharya, Qian, and Yang (2016) study shadow banking in China but, unlike us, they do not explore how stricter liquidity regulation can lead to a credit boom, the thrust of our contribution. Instead, they focus on the rise of Chinese shadow banking and emphasize non-regulatory factors such as contractionary monetary policy and fiscal stimulus. We will present facts and evidence on loan-to-deposit enforcement having been the trigger for shadow banking activities in China and discuss the relevance of alternative explanations in our empirical and quantitative sections and the related appendices. Other pertinent papers on this topic include Allen, Qian, Tu, and Yu (2015) who provide a fairly sanguine picture of Chinese shadow banking outside of wealth management products, Chen, He, and Liu (2016) who find that shadow banking may have helped refinance stimulus loans well after wealth management products developed, and Wang, Wang, Wang, and Zhou (2016) who argue that the Chinese government tacitly accepted these products as a form of interest rate liberalization.

The rest of our paper is organized as follows. Sections 2 and 3 focus on the model. To help isolate the effect of interbank market structure, Section 2 lays out a benchmark model with only small banks and studies the equilibrium properties. Section 3 extends the benchmark to include a large bank, studies how the equilibrium properties are affected, and presents the main analytical results. All proofs are in Appendix A. Sections 4 and 5 then apply the model to China. Section 4 explains why the basic features of China's banking system are well captured by our extended model and provides empirical support for the model's cross-sectional predictions. Calibration results are presented in Section 5 along with a structural estimation that decomposes the importance of various shocks. Section 6 concludes.

2 Benchmark Model

There are three periods, $t \in \{0, 1, 2\}$, and a unit mass of risk neutral banks, $j \in [0, 1]$. Let X_j denote the funding obtained by bank j at $t = 0$. Each bank can invest in a project which returns $(1 + i_A)^2$ per unit invested. Projects are long-term, meaning that they run from $t = 0$ to $t = 2$ without the possibility of liquidation at $t = 1$. To introduce a tradeoff between investing and not investing, banks are also subject to short-term idiosyncratic liquidity shocks which must be paid off at $t = 1$. More precisely, bank j must pay $\theta_j X_j$ at $t = 1$ in order to continue operation. The exact value of θ_j is drawn from a two-point distribution:

$$\theta_j = \begin{cases} \theta_\ell & prob.\ \pi \\ \theta_h & prob.\ 1-\pi \end{cases}$$

where $0 < \theta_\ell < \theta_h < 1$ and $\pi \in (0, 1)$. Each bank learns the realization of its shock in $t = 1$. Prior to that, only the distribution is known.

2.1 Bank Liabilities

The liquidity shocks just described can be fleshed out using Diamond and Dybvig (1983). Specifically, the economy has an endowment $X > 0$ at $t = 0$ and banks attract funding by offering liquidity services to the owners of this endowment (households). The liquidity service offers households more than the long-term project if liquidated at $t = 1$ but less than this project if held until $t = 2$. The traditional liquidity service is a deposit. To set notation, a dollar deposited at $t = 0$ becomes $1 + i_B$ if withdrawn at $t = 1$ and $(1 + i_D)^2$ if withdrawn at $t = 2$. In Diamond and Dybvig (1983), banks choose i_B and i_D to achieve optimal risk-sharing for households. In Diamond and Kashyap (2015), banks take i_B and i_D as given. For the analytical results, we normalize $i_B = i_D = 0$ so that traditional deposits are equivalent to storage. However, each bank j can choose to offer an alternative liquidity service which delivers storage plus a return ξ_j. We will refer to this alternative as a deposit-like product or DLP. To ease the exposition, suppose ξ_j accrues at $t = 2$. As we will explain in Section 2.4, bank j chooses ξ_j to maximize its expected profit subject to household demand for liquidity services. If bank j optimally sets $\xi_j = 0$, then it is content offering storage. The shock θ_j represents the fraction of households that withdraw deposits and DLPs from bank j at $t = 1$.

We now need to specify how households allocate their endowment at $t = 0$ conditional on interest rates. Let D_j denote the funding attracted by bank j in the form of traditional deposits. The funding attracted in the form of DLPs is denoted by W_j, with $X_j \equiv D_j + W_j$ and $\int X_j dj = X$. Appendix B sketches a simple household optimization problem with transactions costs which motivates the following functional forms:

$$W_j = \omega \xi_j \tag{1}$$

$$D_j = X - (\omega - \rho)\, \xi_j - \rho \overline{\xi} \tag{2}$$

where ω and ρ are non-negative constants and $\overline{\xi}$ denotes the average DLP return offered by other banks. Intuitively, ω captures the substitutability between liquidity services within a bank while ρ governs the intensity of competition among banks. To see this, sum equations (1) and (2) to write bank j's funding share as:

$$X_j = X + \rho\left(\xi_j - \overline{\xi}\right) \tag{3}$$

If $\rho = 0$, then bank j perceives its funding share as fixed, shutting down competition. If $\rho > 0$, then bank j perceives a positive relationship between its funding share and the DLP return it offers relative to other banks.

Each individual bank will take $\overline{\xi}$ as given when making decisions. In a symmetric equilibrium, $\overline{\xi}$ will be such that the profit-maximizing choice of ξ_j equals $\overline{\xi}$ for all j.

2.2 Bank Assets and the Interbank Market

We now elaborate on how banks allocate their funding. The maturity mismatch between investment projects and liquidity shocks introduces a role for reserves (i.e., savings which can be used to pay realized liquidity shocks). As we will explain in Section 2.4, the division of X_j into investment and reserves is chosen at $t = 0$ to maximize expected profit.

Let $R_j \in [0, X_j]$ denote the reserve holdings of bank j at $t = 0$. If $\theta_j < \frac{R_j}{X_j}$, then bank j has a reserve surplus at $t = 1$. If $\theta_j > \frac{R_j}{X_j}$, then bank j has a reserve shortage at $t = 1$. An interbank market exists at $t = 1$ to redistribute reserves across banks. A market in which banks can share risk and obtain liquidity also exists in Allen and Gale (2004). The interbank interest rate in our benchmark is denoted by i_L. Banks in the continuum are atomistic so they take i_L as given when making decisions. However, i_L is endogenous and adjusts to clear the interbank market. Interbank lenders (borrowers) are banks with reserve surpluses (shortages) at $t = 1$. In practice, central banks also serve as lenders of last resort so we introduce a supply of external funds, $\Psi(i_L) \equiv \psi i_L$, where $\psi > 0$. We will focus on symmetric equilibrium, in which case R_j and ξ_j are the same across the unit mass of banks. Notice that symmetry of ξ_j in equation (3) implies $X_j = X$. The condition for interbank market clearing is then:

$$R_j + \psi i_L = \overline{\theta} X \tag{4}$$

where $\overline{\theta} \equiv \pi\theta_\ell + (1-\pi)\theta_h$ is the average liquidity shock. Total credit in this economy is the total amount of funding invested in projects (i.e., $X - R_j$).

2.3 Liquidity Regulation and Possible Arbitrage

We now allow for the possibility of a government-imposed loan limit on each bank. This limit can also be viewed as a liquidity rule which says the ratio of reserves to funding must be at least $\alpha \in (0,1)$. Given the structure of our model, reserves are meant to be used at $t = 1$ so enforcement of the liquidity rule is confined to $t = 0$. If the government does not enforce a liquidity rule, then $\alpha = 0$.

Importantly, the liquidity rule only applies to activities that the bank reports on its balance sheet. To model this, we allow banks to choose where to manage DLPs and the projects financed by those DLPs. If fraction $\tau_j \in [0,1]$ is managed in an off-balance-sheet vehicle, then bank j's reserve holdings only need to satisfy:

$$R_j \geq \alpha \left(X_j - \tau_j W_j\right) \tag{5}$$

The off-balance-sheet vehicles in our model capture accounting maneuvers that banks can use to shift activities away from regulation without changing the nature of those activities. This constitutes regulatory arbitrage.[3] If bank j chooses $\xi_j > 0$ and $\tau_j = 0$, then it is simply offering a deposit with a competitive interest rate. If it chooses $\xi_j > 0$ and $\tau_j > 0$, then it is offering this product in order to lessen the burden of the liquidity rule.

2.4 Optimization Problem of Representative Bank

The expected profit of bank j at $t = 0$ is:

$$\Upsilon_j \equiv (1 + i_A)^2 (X_j - R_j) + (1 + i_L) R_j - \left[i_L \overline{\theta} X_j + X_j + \left(1 - \overline{\theta}\right) \xi_j W_j\right] - \frac{\phi}{2} X_j^2 \tag{6}$$

where W_j and X_j are given by (1) and (3) respectively. The first term in (6) is revenue from investment. The second term is revenue from lending reserves on the interbank market. The third term is the bank's expected funding cost, namely the expected cost of borrowing reserves on the interbank market and the expected payments to households. The fourth term is a general operating cost (with $\phi > 0$) which is quadratic in the bank's funding share. Operating costs will play a minimal role until Section 3.

The representative bank chooses the attractiveness of its DLPs ξ_j, the intensity of its

[3] Adrian, Ashcraft, and Cetorelli (2013) define regulatory arbitrage as "a change in structure of activity which does not change the risk profile of that activity, but increases the net cash flows to the sponsor by reducing the costs of regulation." In principle, we could introduce a small cost to pursuing the accounting maneuvers that permit regulatory arbitrage. We do not do this here as it would clutter the exposition without producing much additional insight.

off-balance-sheet activities $\tau_j \in [0,1]$, and its reserve holdings R_j to maximize Υ_j subject to the liquidity rule in (5). The Lagrange multiplier on (5) is the shadow cost of reserves. We denote it by μ_j. The multipliers on $\tau_j \geq 0$ and $\tau_j \leq 1$ are denoted by η_j^0 and η_j^1 respectively. The first order conditions with respect to R_j, τ_j, and ξ_j are then:

$$\mu_j = (1+i_A)^2 - (1+i_L) \tag{7}$$

$$\eta_j^1 = \eta_j^0 + \alpha\mu_j W_j \tag{8}$$

$$\xi_j = \underbrace{\frac{(1-\overline{\theta})\, i_L + (1-\alpha)\,\mu_j - \phi X_j}{2\,(1-\overline{\theta})} \times \frac{\rho}{\omega}}_{\text{competitive motive}} + \underbrace{\frac{\alpha\mu_j}{2\,(1-\overline{\theta})} \times \tau_j}_{\text{reg. arbitrage motive}} \tag{9}$$

The first term on the right-hand side of equation (9) captures what we will call the competitive motive for DLP issuance. If this term is positive, then bank j wants to offer higher DLP returns in order to attract more funding. Recall that bank j's total funding, X_j, is given by equation (3). Each bank takes $\overline{\xi}$ as given so increasing ξ_j relative to $\overline{\xi}$ increases X_j. The second term on the right-hand side of equation (9) captures what we will call the regulatory arbitrage motive for DLP issuance. In the absence of a liquidity rule ($\alpha = 0$), there is no regulatory arbitrage motive. There is also no such motive when the interbank rate is high enough to make the shadow cost of reserves (μ_j) zero.

2.5 Results for Benchmark Model

We now study the equilibrium properties of the benchmark model.

To fix ideas, let's start from an equilibrium where banks are content offering only storage (i.e., $\xi_j^* = 0$, where asterisks denote equilibrium values). We have already established that there is no regulatory arbitrage motive for DLP issuance without liquidity regulation ($\alpha = 0$). The following proposition establishes the conditions under which there is also no competitive motive:

Proposition 1 *Suppose $\phi < \overline{\phi}$ where $\overline{\phi}$ is a positive threshold. If $\alpha = 0$, then $\xi_j^* = 0$ if and only if $\rho = 0$.*

With $\rho = 0$, there is no competitive motive for DLP issuance because each bank perceives its funding share as fixed. With $\rho > 0$ and high operating costs ($\phi \geq \overline{\phi}$), there is also no competitive motive because banks do not want to get bigger. Therefore, $\alpha = 0$ with one of these parameterizations delivers an equilibrium where all banks choose to offer only storage.

Suppose the economy is initially in such an equilibrium. Proposition 2 shows that introducing a sufficiently strict liquidity rule – that is, increasing α above a threshold $\widetilde{\alpha}$ – triggers the issuance of off-balance-sheet DLPs. The benchmark model thus delivers a shadow banking sector:

Proposition 2 *Suppose $\rho = 0$. There is a unique $\widetilde{\alpha} \in \left[0, \overline{\theta}\right)$ such that $\xi_j^* = 0$ if $\alpha \leq \widetilde{\alpha}$ and $\xi_j^* > 0$ with $\tau_j^* = 1$ otherwise.*

The incentive to issue DLPs in Proposition 2 does not come from competition since $\rho = 0$ eliminates the competitive motive. Instead, DLPs are issued because they can be booked off-balance-sheet, away from the binding liquidity rule. It is straightforward to show that $\rho > 0$ with ϕ sufficiently high delivers the same intuition as Proposition 2.

Consider now the aggregate effects. Proposition 3 shows that introducing a liquidity minimum into the benchmark model lowers the equilibrium interbank rate. It is then immediate, given equation (4), that total credit $(X - R_j^*)$ also falls.[4] In other words, introducing regulation into the benchmark model with only small banks has the intended effect.

Proposition 3 *For any $\rho \geq 0$, the interbank rate in the benchmark model is highest at $\alpha = 0$. Therefore, moving from $\alpha = 0$ to $\alpha > 0$ will not increase the interbank rate or total credit. Moving from $\alpha = 0$ and $\rho = 0$ to $\alpha > 0$ and $\rho > 0$ also will not increase the interbank rate or total credit.*

Proposition 3 is basically the market mechanism at work. Suppose there is no government intervention ($\alpha = 0$). At low interbank rates, price-taking banks will rely on the interbank market for liquidity instead of holding their own reserves. In a Walrasian market, all banks are price-takers so liquidity demand at $t = 1$ will exceed liquidity supply. This cannot be an equilibrium. Therefore, the interbank rate must be high to substitute for government intervention.[5]

3 Full Model: Heterogeneity in Market Power

We now extend the benchmark model to include a big bank. By definition of being big, this bank will internalize how all of its choices affect the equilibrium.

We keep the continuum of small banks, $j \in [0, 1]$, and index the big bank by k. DLP demands are $W_j = \omega \xi_j$ and $W_k = \omega \xi_k$, similar to equation (1). The funding attracted by each bank is an augmented version of equation (3), namely:

$$X_j = 1 - \delta_0 + (\delta_1 + \delta_2)\, \xi_j - \delta_1 \xi_k - \delta_2 \overline{\xi_j} \tag{10}$$

[4] With $\psi = 0$ in (4), total credit would be constant. Either way then, there cannot be a credit boom.

[5] See Farhi, Golosov, and Tsyvinski (2009) for a different environment in which a liquidity floor decreases interest rates.

$$X_k = \delta_0 + \delta_1 \left(\xi_k - \overline{\xi_j}\right) \tag{11}$$

where total funding in the economy has been normalized to $X = 1$ and $\overline{\xi_j}$ is the average return on small bank DLPs. Here, δ_1 is the competition parameter between the big and small banks while δ_2 affects the competition among small banks. Small banks take $\overline{\xi_j}$ and ξ_k as given, along with being interbank price-takers. In a symmetric equilibrium, the profit-maximizing choice of ξ_j equals $\overline{\xi_j}$.

The big bank does not take $\overline{\xi_j}$ as given. It is also not an interbank price-taker. As a result, the interbank rate will depend on the big bank's realized liquidity shock. This makes the big bank's shock an aggregate shock so, in Appendix C, we show that adding aggregate shocks to the benchmark model with only small banks does not change Proposition 3.

Let i_L^s denote the interbank rate when the big bank realizes θ_s at $t = 1$, where $s \in \{\ell, h\}$. The interbank market clearing condition for $s = h$ is:

$$R_j + R_k + \psi i_L^h = \overline{\theta} X_j + \theta_h X_k \tag{12}$$

The left-hand side captures the supply of liquidity while the right-hand side captures the demand for liquidity, in an equilibrium where small banks are symmetric. All decisions are made at $t = 0$ so, to convey our main points, it will be enough for the big bank to affect the expected interbank rate, $i_L^e \equiv \pi i_L^\ell + (1 - \pi)\, i_L^h$. We can therefore simplify the exposition by fixing $i_L^\ell = 0$ and letting i_L^e move with i_L^h, where i_L^h is determined as above. It will be verified in the proofs that $i_L^\ell = 0$ does not result in a liquidity shortage when the big bank realizes $\theta_\ell < \theta_h$ in this class of equilibria.

3.1 Optimization Problem of Big Bank

At $t = 0$, the big bank's expected profit is:

$$\Upsilon_k \equiv (1 + i_A)^2 (X_k - R_k) + \left[1 + (1 - \pi)\, i_L^h\right] R_k - \left[(1 - \pi)\, i_L^h \theta_h X_k + X_k + \left(1 - \overline{\theta}\right) \omega \xi_k^2\right] - \frac{\phi}{2} X_k^2$$

The interpretation is similar to equation (6): the first term is revenue from investment, the second term is the potential expected revenue from lending reserves, the third term is the big bank's expected funding cost, and the fourth term is an operating cost.

The big bank chooses R_k, τ_k, and ξ_k to maximize Υ_k subject to three sets of constraints. First are the aggregate constraints, namely funding shares as per (10) and (11) and market clearing as per (12). The market clearing equation connects R_k and i_L^h so saying that the big bank chooses R_k with i_L^h determined by (12) is equivalent to saying that it chooses i_L^h

with R_k determined by (12). This is the sense in which the big bank is a price-setter on the interbank market.

The second set of constraints are the first order conditions of small banks. The representative small bank solves essentially the same problem as before: its objective function is still given by (6) but with $(1-\pi)\, i_L^h$ as the interbank rate and X_j as per equation (10).

The last set of constraints on the big bank's problem are inequality constraints, namely the liquidity rule and non-negativity conditions:

$$R_k \geq \alpha\,(X_k - \tau_k W_k)$$

$$\tau_k \in [0,1]$$

$$\xi_k \geq 0$$

$$\mu_j \geq 0$$

where μ_j is the shadow cost of reserves or, equivalently, the Lagrange multiplier on the liquidity rule in the small bank problem. Each inequality constraint listed above can be either binding or slack.

3.2 Results for Full Model

An equilibrium in the full model is characterized by the first order conditions from the small bank problem, the first order conditions from the big bank problem, and interbank market clearing.

Following Section 2.5, let's start from an equilibrium where all banks offer only storage. We know from our analysis of the benchmark model that small banks will have a competitive motive for DLP issuance if (i) they do not perceive their funding shares as fixed and (ii) operating costs are low enough that they want to expand. Notice that $\delta_1+\delta_2>0$ in equation (10) furnishes (i). If instead $\delta_1+\delta_2=0$, then small banks take their funding shares as given[6] and the first order conditions from their optimization problem deliver:

$$\mu_j\left[R_j - \alpha\left(X_j - \omega\xi_j\right)\right] = 0 \text{ with complementary slackness} \tag{13}$$

$$\mu_j = (1+i_A)^2 - \left[1+(1-\pi)\, i_L^h\right] \tag{14}$$

[6] With $\delta_1+\delta_2=0$, equation (10) becomes $X_j = 1-\delta_0+\delta_1\left(\overline{\xi_j}-\xi_k\right)$. In a symmetric equilibrium, $\xi_j = \overline{\xi_j}$ so there is still an indirect effect of ξ_j on X_j. The point is that small banks are not setting ξ_j to exploit this effect.

$$\xi_j = \frac{\alpha\mu_j}{2\left(1-\overline{\theta}\right)} \tag{15}$$

In words, these equations say that small banks are content offering only storage unless there is a liquidity rule ($\alpha > 0$) and a shadow cost to holding reserves ($\mu_j > 0$). With $\alpha > 0$ and $\mu_j > 0$, small banks would also offer off-balance-sheet DLPs, which is the same regulatory arbitrage motive for DLP issuance seen in equations (8) and (9) of the benchmark model.[7] Clearly, $\alpha = 0$ will be enough to deliver an initial equilibrium without regulatory arbitrage so that small banks do indeed offer only storage at the combination of $\alpha = 0$ and $\delta_1 + \delta_2 = 0$.

To simplify the analytical exposition and develop clear intuition, this section will study a move from $\alpha = 0$ to $\alpha > 0$ assuming $\delta_1 + \delta_2 = 0$. In Section 5.1, we will calibrate the starting and ending values of α to data and allow $\delta_1 + \delta_2 > 0$. We will then calibrate an operating cost parameter for small banks (ϕ_j) that is consistent with no DLP issuance in the initial steady state.[8]

The property that the big bank also offers only storage when $\alpha = 0$ can be delivered in one of two ways as well. One approach is to set $\delta_1 = 0$ in equation (11) so that the big bank's funding share is fixed at $X_k = \delta_0$. Another approach is to keep the big bank's funding share endogenous (i.e., $\delta_1 > 0$) but set a sufficiently high operating cost parameter which eliminates any incentive for the big bank to increase its funding share (and hence issue DLPs) at the configuration of parameters in the initial equilibrium. We will present analytical results for both approaches to isolate how, if at all, an endogenous funding share affects the big bank's decision-making. When considering the second approach in the analytical results below, we will set ϕ so that, in the initial equilibrium, ξ_k is exactly zero as opposed to being constrained by zero. The numerical results in Section 5.1 will also follow the second approach, calibrating ϕ_k for the big bank to distinguish it from the ϕ_j for small banks mentioned above.

Having explained the defining features of the initial equilibrium, let's consider the distribution of reserves between big and small banks in this equilibrium. This was not a consideration in the benchmark model because all banks were ex ante identical price-takers. Now the big bank is a price-setter so its reserve choices may differ from that of small banks.

Proposition 4 *Suppose $\alpha = 0$. Consider $\delta_1 + \delta_2 = 0$ and either $\delta_1 = 0$ or $\delta_1 > 0$ with ϕ sufficiently positive so that the initial equilibrium has $\xi_j^* = \xi_k^* = 0$. If i_A lies within an intermediate range, then the initial equilibrium also involves $\mu_j^* > 0$, $R_j^* = 0$, and $R_k^* > 0$.*

[7] See the proof of Proposition 2 for further discussion.

[8] Notice that the approach in the calibration imposes weaker conditions than $\delta_1 + \delta_2 = 0$: the latter means that there is never a competitive motive among small banks while the former just means that there is no competitive motive at the parameters of the initial equilibrium. The main qualitative results do not depend on which approach is used.

Proposition 4 says that reserves in the initial equilibrium are held disproportionately by the big bank when the returns to investment (i_A) are moderate. The big bank's willingness to hold liquidity reflects its status as an interbank price-setter. In particular, the big bank understands that not holding enough liquidity will increase its funding costs should it experience a high liquidity shock. In contrast, the price-taking small banks invest all their funding in projects and rely on the interbank market, which now includes the big bank, to honor short-term obligations.

We saw in Section 2.5 that introducing a liquidity minimum into the benchmark model with only small banks decreased both the interbank rate and the total amount of credit. In other words, regulation had the intended effect. We want to see whether this is still the case in the full model with big and small banks or whether there are conditions under which the result is reversed. To make the policy experiment concrete, suppose the government moves from $\alpha = 0$ to $\alpha = \overline{\theta}$. As shown next, introducing a liquidity minimum into the full model can lead to an increase in the interbank rate, in sharp contrast to the benchmark prediction:

Proposition 5 *Keep $\delta_1 + \delta_2 = 0$ as in Proposition 4. The following are sufficient for $\alpha = \overline{\theta}$ to generate higher i_L^{h*} than $\alpha = 0$ while preserving slackness of the big bank's liquidity rule ($R_k^* > \alpha X_k^*$), bindingness of the small bank liquidity rule ($\mu_j^* > 0$), and feasibility of $i_L^{\ell*} = 0$:*

1. *Suppose $\delta_1 = 0$ so that the big bank's funding share is fixed. The sufficient conditions are: π sufficiently high, θ_ℓ and $\frac{\psi}{\omega}$ sufficiently low, and i_A within an intermediate range.*

2. *Suppose $\delta_1 = \omega > 0$ so that the big bank's funding share is endogenous. Also set ϕ so that ξ_k^* is exactly zero at $\alpha = 0$ for the reasons discussed at the beginning of this section. The sufficient conditions are: π sufficiently high, θ_ℓ and $\frac{\psi}{\omega}$ sufficiently low, and i_A and δ_0 within intermediate ranges.*

There is a non-empty set of parameters satisfying the sufficient conditions in both 1 and 2. All else constant, the model with an endogenous funding share generates a larger increase in the interbank rate than the model with a fixed funding share.

We devote Section 3.2.1 to explaining the interest rate results just established. Section 3.2.2 will then establish several other results that distinguish the full model from the benchmark, including the effect of liquidity regulation on total credit.

3.2.1 Intuition for Interest Rate Response

To explain Proposition 5, it will be useful to summarize all the forces behind the big bank's choice of i_L^h. Differentiating the big bank's objective function with respect to i_L^h:

$$\frac{\partial \Upsilon_k}{\partial i_L^h} \propto \underbrace{R_k - \theta_h X_k}_{\text{direct motive}} - \underbrace{\left[\frac{(1+i_A)^2 - 1}{1-\pi} - i_L^h\right]\frac{\partial R_k}{\partial i_L^h}}_{\text{reallocation motive}} + \underbrace{\left[\frac{(1+i_A)^2 - 1 - \phi X_k}{1-\pi} - \theta_h i_L^h\right]\frac{\partial X_k}{\partial i_L^h}}_{\text{funding share motive}} \tag{16}$$

The equilibrium i_L^h solves $\frac{\partial \Upsilon_k}{\partial i_L^h} = 0$ when the relevant inequality constraints in the big bank's problem are slack. This is the appropriate case given the statement of Proposition 5. We will start by explaining the three motives identified in (16). We will then explain how the strength of each motive varies with α in order to understand why moving from $\alpha = 0$ to $\alpha = \overline{\theta}$ generates a higher interbank rate.

First is the direct motive. The big bank has reserves R_k and a funding share X_k. Its net reserve position when hit by a high liquidity shock is therefore $R_k - \theta_h X_k$. Each unit of reserves is valued at an interest rate of i_L^h when the big bank's shock is high so, on the margin, an increase in i_L^h changes the big bank's profits by $R_k - \theta_h X_k$.

Second is the reallocation motive. The idea is that changes in i_L^h also affect how many reserves the big bank needs to hold in a market clearing equilibrium. If $\frac{\partial R_k}{\partial i_L^h} < 0$, then an increase in i_L^h elicits enough liquidity from other sources to let the big bank reallocate funding from reserves to investment. On the margin, the value of this reallocation is the shadow cost of reserves, hence the coefficient on $\frac{\partial R_k}{\partial i_L^h}$ in (16).

Third is the funding share motive. The idea is that changes in i_L^h also affect how much funding the big bank attracts when funding shares are endogenous. If $\frac{\partial X_k}{\partial i_L^h} > 0$, then an increase in i_L^h curtails the DLP offerings of small banks by enough to boost the big bank's funding share. The coefficient on $\frac{\partial X_k}{\partial i_L^h}$ in (16) captures the marginal value of a higher funding share for the big bank. We will discuss this coefficient in more detail below.

To gain some insight into how changes in α will affect the solution to $\frac{\partial \Upsilon_k}{\partial i_L^h} = 0$ through each motive, let's start with the case of fixed funding shares ($\delta_1 = 0$). Consider first the direct motive. Using the market clearing condition:

$$R_k - \theta_h X_k \overset{\delta_1 = 0}{=} \overline{\theta}(1-\delta_0) - \psi i_L^h - \underbrace{\alpha\left(1 - \delta_0 - \frac{\alpha\omega(1-\pi)}{2\left(1-\overline{\theta}\right)}\left[\frac{(1+i_A)^2 - 1}{1-\pi} - i_L^h\right]\right)}_{R_j \text{ as per small bank FOCs in (13) to (15)}} \tag{17}$$

For a given value of i_L^h, the magnitude of the direct motive in (17) depends on α through the reserve holdings of small banks. There are two competing effects. On one hand, higher α forces small banks to hold more reserves per unit of on-balance-sheet funding. On the other hand, higher α can compel small banks to engage in regulatory arbitrage, decreasing their on-balance-sheet funding as they offer off-balance-sheet DLPs (i.e., $\xi_j > 0$ with $\tau_j = 1$). The net effect is ambiguous so we must look beyond the direct motive to fully understand Proposition 5.

With fixed funding shares, the only other motive is the reallocation motive, where:

$$\left.\frac{\partial R_k}{\partial i_L^h}\right|_{\delta_1=0} = -\psi - \frac{\alpha^2\omega\,(1-\pi)}{2\left(1-\overline{\theta}\right)} < 0 \tag{18}$$

This expression is negative for two reasons. First and as captured by the first term in (18), a higher interbank rate will attract more external liquidity, allowing the big bank to hold fewer reserves. Second and as captured by the second term in (18), small banks will increase their reserves when the interbank rate increases, also allowing the big bank to hold fewer reserves. The effect of i_L^h on R_j that underlies the second term works through the regulatory arbitrage motive of small banks: there is less incentive to circumvent a liquidity regulation when the price of liquidity is expected to be high. We can also infer from the second term that the effect of i_L^h on R_j strengthens with α. This is both because R_j is more responsive to changes in ξ_j at high α (see equation (13)) and because ξ_j is more responsive to changes in i_L^h at high α (see equations (14) and (15)).

This discussion helps explain the first bullet in Proposition 5: when funding shares are fixed, high α makes it easier for the big bank to use high interbank rates to incent small banks to share the burden of keeping the system liquid.

Does the same intuition extend to the case of endogenous funding shares? No because:

$$\left.\frac{\partial R_k}{\partial i_L^h}\right|_{\delta_1=\omega} = -\psi + \frac{\alpha\omega\pi\,(\theta_h-\theta_\ell)\,(1-\pi)}{2\left(1-\overline{\theta}\right)} \tag{19}$$

An increase in i_L^h still decreases ξ_j but now a decrease in ξ_j also decreases how much funding small banks attract (X_j) and therefore how many reserves they will want to hold.[9] This effect is strong enough to make the second term in (19) positive, in contrast to the second term in (18).

We must therefore turn to the funding share motive to fully understand why introducing a liquidity minimum can increase the equilibrium interbank rate when funding shares are

[9] See footnote 6 for the effect of ξ_j on X_j when $\delta_1 + \delta_2 = 0$.

endogenous. Recall the expression for the funding share motive from (16) and note:

$$\left.\frac{\partial X_k}{\partial i_L^h}\right|_{\delta_1=\omega} = \frac{\alpha\omega\left(1-\pi\right)}{2\left(1-\overline{\theta}\right)} > 0 \tag{20}$$

We already know from the discussion of (19) that an increase in i_L^h decreases ξ_j which then decreases the small bank funding share X_j. Total funding is normalized to one so the decrease in X_j implies an increase in the big bank funding share X_k. The expression in (20) is therefore positive. The magnitude of this expression increases with α because ξ_j is more responsive to changes in i_L^h at high α (see equations (14) and (15)). It is therefore easier for the big bank to increase its funding share by increasing i_L^h when α is high.

There is, of course, a difference between the ability to increase funding share and the desire to do so. To complete the intuition, let us reconcile the big bank's desire to increase its funding share when α is high with the existence of convex operating costs. Return to the coefficient on $\frac{\partial X_k}{\partial i_L^h}$ in (16). All else constant, moving from $\alpha = 0$ to $\alpha = \overline{\theta}$ will trigger regulatory arbitrage by small banks. The presence of $\xi_j > 0$ will then erode the big bank's funding share X_k, lowering its marginal operating cost ϕX_k.

We can now understand the second bullet in Proposition 5 as follows: when funding shares are endogenous, high α makes it easier for the big bank to use high interbank rates to stop small banks from encroaching on its funding share. The last part of Proposition 5 establishes that sizeable increases in the interbank rate are most consistent with this sort of asymmetric competition, wherein the big bank uses its interbank market power to fend off competition from small banks and their off-balance-sheet activities.

3.2.2 Credit Boom and Cross-Sectional Predictions

We have now explained how the full model can deliver an increase in the equilibrium interbank rate when a liquidity minimum is introduced. Proposition 6 below shows that the full model also delivers an increase in total credit $(1 - R_j^* - R_k^*)$ after the introduction of this regulation. In other words, unlike the benchmark model, regulation in the full model can be entirely counterproductive. Proposition 6 also summarizes the cross-sectional implications of introducing liquidity regulation into the full model, namely convergence of on-balance-sheet ratios and more aggressive issuance of DLPs by small banks relative to the big bank (i.e., $\xi_j^* > \xi_k^*$). Under the parameter conditions in Proposition 5, which are also the parameter conditions in Proposition 6, the small bank liquidity rule is binding and the big bank liquidity rule is slack. Therefore, small banks issue all their DLPs off-balance-sheet ($\tau_j^* = 1$) while the big bank is indifferent between any $\tau_k^* \in [0, 1]$. We use $\tau_k^* = 0$ in the proofs to fix ideas but the general point given the indifference of the big bank is that small banks are more squarely involved in off-balance-sheet activity.

Proposition 6 *Invoke the parameter conditions from Proposition 5 and define the loan-to-deposit ratio as the ratio of on-balance-sheet investment to on-balance-sheet funding. Total credit increases and the loan-to-deposit ratios of big and small banks converge when we move from $\alpha = 0$ to $\alpha = \overline{\theta}$. Moreover, $\xi_j^* > \xi_k^*$ at $\alpha = \overline{\theta}$, with $\xi_k^* > 0$ if and only if funding share is endogenous. This is in contrast to $\xi_j^* = \xi_k^* = 0$ at $\alpha = 0$.*

It may now be useful to recap the intuition for our results, highlighting along the way the channels through which total credit increases. Small banks move into off-balance-sheet DLPs after liquidity rules tighten. As they push to attract funding into these products, they offer interest rates above the rates on traditional deposits. All else constant, this poaches deposits from the big bank. Recall that the big bank internalizes the effect of reserve holdings on the interbank market. Therefore, compared to small banks, it invests less at $t = 0$ per unit of funding attracted. The reallocation of funding from deposits at the big bank to high-return DLPs at the small banks thus increases total credit. This is one of two channels.

The second channel stems from how the big bank responds to its loss of funding. One way for the big bank to respond is by offering its own DLPs with high interest rates. Naturally, this is costly because of the high rates. Another way for the big bank to respond is to use the interbank market. Small banks have less incentive to skirt liquidity rules if they expect the price of liquidity to be high. All else constant, the interbank market at $t = 1$ will be less liquid and the expected interbank rate will rise if the big bank holds fewer reserves at $t = 0$. The big bank can thus change interbank market conditions to make small banks scale back their issuance of DLPs. While this strategy by the big bank curbs *some* of the increase in total credit from the first channel, it also boosts credit directly because the big bank shifts from reserves to investment at $t = 0$. The big bank's strategy also contributes directly to a rise in its loan-to-deposit ratio. This is one part of the convergence result in Proposition 6. The other part is that the small bank ratio, as measured *on balance sheet*, falls to comply with the liquidity rule.

4 Application of Full Model to China

We have focused so far on qualitative predictions of the theory. We now want to study quantitative implications. We choose China, one of the world's largest and most rapidly growing economies, as the setting for our quantitative analysis. Because of its stage of development, China's banking system has elements in common with both emerging and advanced economies, making the lessons informative for a wider range of countries. Moreover, China has experienced unprecedented growth in private credit relative to private savings over the past decade. Our model predicts that some credit booms are caused by stricter liquidity regulation so we are interested to know whether stricter liquidity regulation can account for at least part of the Chinese experience.

Before proceeding to a full quantitative analysis in Section 5, we establish a mapping from the model to China. Section 4.1 provides some institutional background on the Chinese system and discusses the policies that amount to a tightening of liquidity regulation. We then turn to the data. Propositions 5 and 6 showed that several other changes may accompany a credit boom after stricter liquidity regulation. First is convergence of banks' on-balance-sheet loan-to-deposit ratios, with large banks' ratios increasing and small banks' ratios decreasing. Second is emergence of deposit-like products that offer elevated rates of return, with more aggressive provision of such products by small banks than by large banks and much more off-balance-sheet accounting of such products by small banks than by large banks. Third is restrictive behavior on the interbank market by large banks along with a higher average interbank interest rate and a larger gap between peak and trough interest rates. Section 4.2 documents the occurrence of the first phenomenon in China, Section 4.3 the second, and Section 4.4 the third. Section 4.3 also presents direct evidence that small banks responded to stricter liquidity regulation while large banks responded to the activities of small banks. Our primary dataset is the Wind Financial Terminal. In cases where Wind is insufficient, we collect data from bank annual reports, regulatory agencies, and financial association websites.

4.1 Institutional Background

This section describes the basic features of China's banking system and how they map into our framework. We refer the reader to Appendix D for a more detailed discussion of the reforms and regulations summarized here.

The Chinese economy is served by both big and small banks. Until the late 1970s, China had a Soviet-style financial system where the central bank was the only bank. The Chinese government moved away from this system in the late 1970s and early 1980s by establishing four commercial banks (the Big Four). Market-oriented reforms initiated in the 1990s made the Big Four much more profit-driven and opened the banking sector up to entry by small and medium-sized commercial banks. China now has twelve joint-stock commercial banks (JSCBs) operating nationally and over two hundred city banks operating in specific regions.

Many rural banks have also emerged. The JSCBs are typically larger than the city and rural banks but all of these banks are still individually small when compared to the Big Four. Although the Chinese government dramatically loosened its grip on the Big Four as part of the market-oriented reforms, a legacy expectation of minimal competition between these four banks remains. China's banking sector can therefore be approximated by one big bank and many small banks. This maps into the environment of our full model.

Further reform by the government in 2005 expanded the range of financial services banks could provide. This led to the advent of wealth management products or WMPs for short. WMPs are well approximated by the DLPs in our model. In particular, WMPs represent a liquidity service provided by banks at endogenous interest rates. Banks can also choose where to report their WMPs by choosing whether or not to provide an explicit principal guarantee. Any WMPs issued with an explicit principal guarantee must be reported on-balance-sheet. Absent such a guarantee, the WMP and the assets it invests in do not have to be consolidated into the bank's balance sheet. These unconsolidated WMPs are invested off-balance-sheet, typically with the help of a lightly-regulated financial institution called a trust company. A more detailed discussion of trust companies and their involvement with WMPs appears in Appendix D. The lack of explicit guarantees on off-balance-sheet WMPs is only for accounting purposes though: there is a general perception that all WMPs are at least implicitly guaranteed by traditional banks (see also Elliott, Kroeber, and Qiao (2015)).

Turning to the regulatory environment, China's banks are currently regulated by the People's Bank of China (central bank) and the China Banking Regulatory Commission (CBRC). A loan-to-deposit cap was introduced in 1995 to prevent banks from lending more than 75% of the value of their deposits to non-financial borrowers. However, enforcement was lax until 2008 when CBRC moved to curb lending and bolster liquidity, particularly at small and medium-sized banks.[10] CBRC began by policing the end-of-year loan-to-deposit ratios of all banks more carefully. It then switched to end-of-quarter ratios in late 2009, end-of-month ratios in late 2010, and average daily ratios in mid-2011. Our sample ends in 2014, at which point CBRC was still monitoring average daily ratios. The 2008-2014 period in China thus involved stricter liquidity regulation than the years just prior. Stricter enforcement of the loan-to-deposit cap by CBRC during this period was also complemented by a rapid increase in the reserve requirements set by the central bank.

Banks in China are also subject to capital requirements and deposit rate regulation. Capital requirements follow the international Basel Accords and were a non-binding constraint on Chinese banks for the period of interest. Deposit rate regulations involved the central

[10] As we will establish in Section 4.2, these banks had higher loan-to-deposit ratios than the Big Four, consistent with the predictions of our model.

bank essentially setting deposit rates until late 2015. Our model allows for a liquidity service with exogenous interest rates. Recall that we have storage with $i_B = i_D = 0$. In the calibration, we can upgrade storage to a traditional deposit which has $i_B > 0$ and $i_D > 0$ as set by the government. DLPs, if offered in equilibrium, will pay an additional return relative to these positive rates. Deposit rate regulation in China only applies to traditional deposits so, like the DLPs in our model, WMPs have endogenously determined interest rates.

4.2 Convergence of Loan-to-Deposit Ratios

Figure 1 plots on-balance-sheet loan-to-deposit ratios for the Big Four and the JSCBs from 2005 to 2014.[11] There are three major takeaways. First, the JSCBs have historically had higher loan-to-deposit ratios than the Big Four, as predicted by the theory.[12] Second, stricter enforcement of a 75% loan-to-deposit cap starting around 2008 constrains the JSCBs but not the Big Four. This is observed most clearly by looking at ratios based on average balances during the year rather than end-of-year balances. As a group, the JSCBs were just satisfying the 75% cap in terms of end-of-year loan-to-deposit ratios when stricter enforcement began. However, using average balances during the year, the JSCBs had loan-to-deposit ratios which well exceeded the 75% cap. This implies that the end-of-year ratios were window-dressed to hit 75% and thus not reflective of the true position of the JSCBs. The enforcement action that began in 2008 sought to impose a 75% cap on the true position of banks so it is in this sense that CBRC imposed a binding constraint on the JSCBs. As CBRC increased its monitoring frequency between 2008 and 2011, the difference between the end-of-year and average balance ratios for JSCBs began to disappear. Also notice that the average balance ratio of the JSCBs reached exactly 75% in 2012, the first full year of average balance monitoring by CBRC. In contrast, there has never been a sizeable difference between the average balance and end-of-year ratios of the Big Four, with both ratios comfortably below 75% when stricter enforcement began.[13]

The third major takeaway from Figure 1 is that the loan-to-deposit ratio of the Big Four has increased towards 75% since the beginning of the enforcement. This increase reflects both higher loan growth and lower deposit growth relative to the pre-enforcement period.

[11] Historical balance sheet data for city and rural banks is spotty, particularly when it comes to average daily balances, so these banks are excluded from the figure.

[12] On this point, small banks in the U.S. have historically also had higher loan-to-deposit ratios than large banks. For example, among nationally chartered banks in the U.S., the difference averages 9 percentage points over the period 1985 to 2000, with small and large banks as defined in FRED data.

[13] A common narrative is that the Chinese government uses individual loan quotas to impose even stricter limits on big banks. In practice though, individual loan quotas in China are negotiable, particularly for the Big Four who have more bargaining power than the JSCBs and the city/rural banks, so the Big Four should still be viewed as choosing their own loan-to-deposit ratios.

From 2005 to 2008, loans and deposits at the Big Four grew at annualized rates of 10.9% and 14.1% respectively. From 2008 to 2014, these rates were 16.7% and 12.3% respectively. China's State Council announced a two-year stimulus package in late 2008 which would have required the banking sector to fund roughly RMB 4 trillion of new investment. Appendix E uses a money multiplier calculation to purge loan and deposit growth of the effects of this package. We find that loans and deposits at the Big Four would have grown at annualized rates of 12.9% and 9.8% respectively from 2008 to 2014 absent the stimulus package. The Big Four loan-to-deposit ratio would have then increased from 0.57 in 2008 to 0.67 in 2014. This is around three-quarters of the actual increase plotted in Figure 1 so big banks in China have become less liquid even taking into account the stimulus package. The period of stricter loan-to-deposit enforcement was therefore accompanied by convergence of the on-balance-sheet loan-to-deposit ratios of big and small banks, with the JSCB ratio decreasing to comply with CBRC's tougher stance and the Big Four ratio increasing beyond what can be explained by stimulus. That such convergence would occur after enforcement imposed a binding constraint on only small banks is as predicted in Proposition 6.

4.3 Evidence from Wealth Management Products

The theory also predicts emergence of deposit-like products with higher interest rates than traditional deposits after a loan-to-deposit cap that binds on only small banks is enforced. Moreover, small banks are predicted to offer higher interest rates on these products than large banks and off-balance-sheet issuance is predicted to be dominated by small banks. We start this section by showing these patterns in the data. We then present empirical evidence on the mechanisms behind these patterns. Recently, a few other papers have also begun using disaggregated data to comment on off-balance-sheet activities in China. We discuss these papers in Appendix D and explain why we believe their results support our conclusions.

4.3.1 Patterns in WMP Issuance

Figure 2 plots the evolution of WMPs in China after these products became legal in 2005. Notice that WMP activity was modest prior to 2008, with WMPs outstanding amounting to less than 2% of GDP in 2006 and 2007. In contrast, the period after 2008 was characterized by rapid growth of WMPs and, by 2014, the amount outstanding stood at nearly 25% of GDP. The spread between annualized returns on 3-month WMPs and the 3-month deposit rate has averaged 1 percentage point since 2008 and nearly 2 percentage points since 2012, with virtually all WMPs delivering above or equal to their promised returns regardless of whether or not an explicit principal guarantee was in place. Similar patterns are observed at other maturities but we highlight 3-month rates since median WMP maturity has been between 2 and 4 months since 2008.

Turning to the cross-section, the Big Four have indeed been less aggressive in WMP issuance than small banks (i.e., JSCBs and smaller). From 2008 to 2014, the realized returns on 3-month WMPs issued by small banks averaged over 30 basis points above the realized returns on 3-month WMPs issued by banks in the Big Four. Moreover, the lowerbounds promised by small banks averaged over 80 basis points above those promised by the Big Four and, by 2014, the average lowerbound promised by small banks was 1.5 times the average lowerbound promised by the Big Four.

The Big Four have also been less involved in booking WMPs off-balance-sheet, as predicted above. Between 2008 and 2014, small banks accounted for 73% of all new WMP batches and issued 57% of their batches without a guarantee while the Big Four issued only 46% of their batches in this way. The gap in non-guaranteed intensity widens in the second half of the sample, with small banks at 62% and the Big Four at 43%. These estimates are based on product counts since Wind does not yet have complete data on the total funds raised by each product. However, using data from CBRC and the annual reports of the Big Four, we estimate that small banks accounted for roughly 64% of non-guaranteed WMP balances outstanding at the end of 2013.[14]

In the model, the patterns just discussed arise because small banks are responding to stricter enforcement of loan-to-deposit rules while big banks are responding to increased competition from small banks. We now provide evidence on these mechanisms, beyond just the patterns the model predicts they generate.

4.3.2 Small Banks Respond to Loan-to-Deposit Rules

A detailed case study will help identify the motives that drive small banks. We consider China Merchants Bank (CMB), one of the twelve JSCBs. Between 2007 and 2013, CMB's loan-to-deposit ratio averaged 82% when calculated using average balances during the year. This is in contrast to 74% when end-of-year balances are used. WMP issuance by CMB increased from RMB 0.1 trillion in 2007 to RMB 0.7 trillion in 2008 before reaching almost RMB 5 trillion in 2013. CMB accounted for only 3% of total banking assets in 2012 but 5.2% of WMPs outstanding at year-end and 17.7% of all WMPs issued during the year.[15]

[14]The entire WMP balance in Bank of China's annual report is described as an unconsolidated balance yet the micro data in Wind includes several guaranteed batches for this bank that would not have matured by the end of 2013. We therefore remove Bank of China and rescale the other banks in the Big Four to back out our 64% estimate for small banks.

[15]Based on data from KPMG, CBRC, and China Merchants Bank.

At the end of both 2012 and 2013, CMB had about 83% of its outstanding WMP balances booked off-balance-sheet and, based on notes to the financial statements, figures for earlier years were at least as high.

We argue that time variation in the maturity of CMB's off-balance-sheet WMPs reveals the importance of loan-to-deposit regulation for the evolution of these products. Figure 3 shows a sizeable drop in the median maturity of CMB's off-balance-sheet products, from just over 4 months in late 2009 to just under 1 month by mid-2011. This drop does not occur for on-balance-sheet WMPs nor is it matched by a drop in the promised annualized yield on off-balance-sheet products. Instead, the drop in CMB's off-balance-sheet maturity coincides with changes in CBRC's monitoring of loan-to-deposit ratios. Recall from Section 4.1 that CBRC focused on the end-of-year ratio until late 2009, the end-of-quarter ratio until late 2010, and the end-of-month ratio until mid-2011. CMB thus shortened the maturity of its off-balance-sheet products as the frequency of CBRC exams increased.

This is significant because shorter maturities can be used to thwart more frequent end-of-period exams. Upon maturity, the principal and interest from an off-balance-sheet WMP are automatically transferred to the buyer's deposit account. A buyer who wants to roll over his investment then contacts his bank to have the transfer reversed. In the time between the transfer and the reversal though, reserves and deposits rise, lowering the loan-to-deposit ratio observed by CBRC.[16] In the first half of 2011, CMB's off-balance-sheet products had a median maturity of just under 1 month which enables the window-dressing that thwarts the end-of-month exams. To make this point more concrete, we look at the maturity of each off-balance-sheet batch relative to its issue date. Approximately 15% of the off-balance-sheet batches issued by CMB between January 2008 and December 2010 would have matured near a month-end. This fraction jumped to 40% in early 2011.

Shortening the maturity of off-balance-sheet WMPs in response to increasingly frequent monitoring of end-of-period loan-to-deposit ratios became futile in mid-2011 as CBRC began monitoring average daily ratios, not end-of-period ratios. Accordingly, Figure 3 shows that

[16] Keeping the automatic deposits as reserves is one approach. Another is to bring loans back on balance sheet in the form of securities which derive their cash flows from the loans. The data suggest that CMB just recorded reserves between 2009 and 2011. The process being discussed here – designing unguaranteed WMPs to automatically deposit before a regulatory check and recording a non-loan as the balancing asset – also sheds light on how the JSCBs window-dressed their end-of-year balance sheets in Figure 1. Of course, this process only sheds light on the post-2008 period when WMP issuance was non-trivial. A common practice pre-2008 was to call loans shortly before a potential inspection (e.g., the end of a calendar year). The bank promises to re-issue these loans within a few weeks and, in the intervening time, borrowers whose loans were called essentially rely on loan sharks. This practice is only feasible if it does not have to occur frequently (i.e., the borrowers cannot afford to rely increasingly on loan sharks). Therefore, in the era of stricter and more frequent loan-to-deposit enforcement, any end-of-period window-dressing was primarily accomplished via the maturity of unguaranteed WMPs.

CMB's median off-balance-sheet maturity has returned to roughly 3 months. The fraction of off-balance-sheet batches set to mature near a month-end has also fallen back below 20%. Similar patterns are not observed for the Big Four. That is, changes in WMP maturity do not track changes in CBRC's monitoring of loan-to-deposit rules when we restrict attention to WMPs issued by banks in the Big Four.

4.3.3 Big Banks Respond to Small Banks

Granger causality tests on WMP issuance in China show that big banks were responding to small banks. Using monthly detrended data on WMP batches between January 2007 and September 2014, the null hypothesis that WMP issuance by small banks does not Granger-cause WMP issuance by the Big Four is rejected at 1% significance, regardless of the detrending method or the number of lags. The opposite hypothesis that WMP issuance by the Big Four does not Granger-cause WMP issuance by the small banks cannot be rejected at 10% significance. Our baseline specification is a VAR with six lags and detrending via HP filter. The p-value for the null hypothesis that small bank issuance does not cause Big Four issuance is 0.002 (χ^2 statistic 21.104). In contrast, the p-value for the null that Big Four issuance does not cause small bank issuance is 0.478 (χ^2 statistic 5.5264). Changing the number of lags can decrease the p-value on the latter null, but not below 0.1. Therefore, the impetus for WMPs in China is indeed coming from small banks.

4.4 Interbank Conditions and Big Four Involvement

The last set of theoretical predictions to evaluate are about the interbank market. Propositions 5 and 6 showed that loan-to-deposit caps can lead to tighter interbank conditions together with a credit boom. Big banks were an important force behind this result: in the benchmark model with only small banks, the introduction of loan-to-deposit regulation always led to a lower interbank interest rate and less credit.

Figure 4 shows that tighter and more volatile interbank markets have indeed accompanied the period of interest. China has both an uncollateralized money market (solid black line) and an interbank repo market (solid gray line). We will focus on the latter since it is much larger than the former. However, both markets have clearly exhibited an upward trend in interest rates since 2009 despite fairly large monetary injections by China's central bank (dashed red line). In the interbank repo market, the average interest rate weighted by transaction volume was 50 basis points higher in 2014 than it was in 2007. The highest weighted average rate observed in daily data also increased by 150 basis points after 2007, reaching an unprecedented 11.6% in mid-2013.

We now want to evaluate the extent to which the Big Four contributed to the increase in interbank rates. The incentives that push big banks to optimally tighten the interbank market in our model need not generate a constantly higher interbank rate: an increase in the average interbank rate is enough to increase small banks' incentives to hold liquidity. The mid-2013 event was certainly relevant in this regard so we obtained transaction-level data from the interbank repo market to study it in detail.

A common narrative in China is that interbank conditions tightened in mid-2013 because the government wanted to discipline the market (e.g., Elliott, Kroeber, and Qiao (2015)). Banks in general experienced some liquidity pressure in early June 2013 as companies withdrew deposits to pay taxes and households withdrew ahead of a statutory holiday.[17] Accordingly, the weighted average interbank repo rate rose from 4.6% on June 3 to 9.3% on June 8 before falling back down to 5.4% on June 17. Most of the seasonal pressures seemed to have subsided yet interbank rates rose again on June 20 after the government indicated it would not inject extra liquidity. The weighted average repo rate hit 11.6%, with minimum and maximum rates of 4.1% and 30% respectively. For comparison, the minimum and maximum rates on June 3 were 3.9% and 5.3% respectively.

An analysis of individual transactions will show whether or not tightness on June 20 was triggered by the government. Our identification strategy makes use of the fact that China has three policy banks which raise money in bond markets to fund economic development projects approved by the central government. The policy banks are not commercial banks and are thus distinct from the Big Four, the JSCBs, and the city/rural banks. Wind data on daily net positions from mid-2009 to mid-2010 reveals that the policy banks were the second largest liquidity providers on the interbank market, second only to the Big Four.[18] The policy banks are essentially agents of the government whereas the Big Four have become much more independent since the market-oriented reforms discussed in Section 4.1. Therefore, if the interbank market tightened in mid-2013 at the hands of the government, the policy banks should have been at least as restrictive as the Big Four.

The transaction-level data show that this was not the case. The policy banks provided a lot of liquidity to the interbank market at fairly low interest rates, to the point that they became the largest net lenders on June 20, 2013. The Big Four, on the other hand, were extremely restrictive, amassing RMB 50 billion of net borrowing by the end of the trading

[17] *The Economist*, "The Shibor Shock," June 22, 2013.

[18] The Wind sample runs from July 2009 to September 2010 for a total of 309 trading days. On the 285 trading days where big banks and policy banks were both net lenders, big banks were the main net lender 93% of the time. Moreover, when big banks were the main net lender, their net lending was 4.2 times that of policy banks. In contrast, when policy banks were the main net lender, their net lending was only 1.6 times that of big banks.

day. Figure 5 illustrates the sharp difference between the Big Four and the policy banks in terms of both quantity and price of liquidity provision on June 20. The top panel illustrates the reluctance of the Big Four to lend while the bottom panel illustrates a sizeable increase in policy bank loans and the more moderate nature of policy bank interest rates.

The bottom panel of Figure 5 also reveals that much of the increase in policy bank lending on June 20 was absorbed by the Big Four. Were big banks borrowing because they really needed liquidity? Two pieces of evidence suggest no. First, the Big Four's ratio of repo lending to repo borrowing was 0.7, with 71% of the loans directed towards small banks. If the Big Four were in dire need of liquidity on June 20, we would expect to see very little outflow. Second, the repo activities of big banks involved a maturity mismatch. Overnight trades accounted for 96% of big bank borrowing but only 83% of big bank lending to small banks. Roughly 80% of policy bank lending to small banks was also at the overnight maturity. If big banks really needed liquidity on June 20, we would expect the maturity of their lending to be closer to the maturity of their borrowing. Instead, it was closer to the maturity offered by policy banks to borrower groups that policy banks and big banks had in common.

Figure 6 shows that big banks also commanded an abnormally high interest rate spread on June 20. In particular, their weighted average lending rate was 266 basis points above their weighted average borrowing rate. This is high relative to other banks: JSCBs and city banks commanded spreads of 113 and 46 basis points respectively. It is also high relative to other days in the sample: on any other day in June 2013, the spread commanded by big banks was between -40 and 58 basis points. Pricing among big banks was also much more uniform than pricing among small banks, both on June 20 and throughout our sample. To this point, we calculate the coefficient of variation (CV) of overnight lending rates offered by banks in different groups and find that the CV among big banks was 61% of the CV among JSCBs and 21% of the CV among city banks on June 20. Averaging over all trading days in June 2013 yields similar figures, namely 62% and 29% respectively.[19]

Taken together, the evidence presented in this section indicates that big banks absorbed policy bank liquidity and intermediated it to small banks at much higher interest rates. As shown in Figure 7, JSCBs paid a lot more for non-policy bank loans on June 20 than they did for policy bank loans. It then stands to reason that JSCBs would have liked a higher share of policy bank lending. Instead, they received 20% of what policy banks lent on June 20, down from an average of 28% over the rest of the month.[20] The situation was similar for

[19] We exclude lending rates charged to policy banks given the proximity of policy banks to the government.

[20] For completeness, the overnight and 7 day maturities shown in Figure 7 were almost 94% of JSCB borrowing on June 20. They were also 100% of JSCB borrowing from policy banks on this date. There were no major differences in the haircuts imposed by policy banks versus other lenders.

city and rural banks: they faced large price differentials between policy and non-policy bank loans yet their share of policy bank lending on June 20 was 22%, well below an average of 47% over the rest of the month.

In sum, the Big Four can and do change interbank conditions to the detriment of their smaller competitors. Figure 4 showed a tightening of the interbank market over the same period as the credit boom that we turn to next. This tightening occurred despite increasing monetary injections by the central bank and our analysis of the most dramatic rise in interbank rates revealed the Big Four as the driving force. All of these findings line up with the theory.

5 Quantitative Analysis

We are now ready to quantify how much of China's credit boom can be generated by stricter liquidity regulation. We first present the calibration results. We then establish the external validity of the calibrated model and use it to decompose the importance of various shocks.

5.1 Calibration Results

The starting point for our calibration is 2007, just prior to China's adoption of stricter liquidity rules. The ending point is 2014. The initial liquidity rule is set to $\alpha = 0.14$ to match the observed loan-to-deposit ratio of JSCBs in 2007. We use the ratio based on average balances rather than end-of-year balances, as per Figure 1. The policy experiment is then an increase from $\alpha = 0.14$ to $\alpha = 0.25$, capturing CBRC's stricter enforcement of the 75% loan-to-deposit cap.

We take the time from $t = 0$ to $t = 2$ to be a quarter. All interest rates are quoted on an annualized basis. We set $(1 + i_D)^2 = 1.026$ to match the average benchmark interest rate of 2.6% for 3-month deposits in China. We set $(1 + i_B)^2 = 1.004$ to match the average benchmark interest rate of 0.4% for demand deposits. The central bank's benchmark interest rate for loans with a maturity of less than six months averages 5.6%. We set $(1 + i_A)^2 = 1.05$ since banks can offer a discount of up to 10% on the benchmark loan rate.[21]

For the low liquidity shock, Proposition 5 points to a small magnitude (θ_ℓ) and a high probability (π) so we use $\theta_\ell = 0$ and $\pi = 0.75$. The parameter conditions in Proposition 5 are relevant for China given the strong empirical support found in Section 4 for the predictions

[21] We are assuming the same return i_A for all banks. In practice, different banks can invest in different sectors but the "risk-adjusted" returns are roughly comparable: while the private sector is more productive than the state sector, lending to the private sector is, at least politically, riskier. Some anecdotal evidence can be found in Dobson and Kashyap (2006).

of Propositions 5 and 6. Proposition 5 also points to a low external liquidity parameter (ψ) so we use $\psi = 0.5$. In words, a one percentage point increase in the interbank rate prompts a monetary injection by the central bank of 0.5% of total savings. This is a reasonable magnitude in light of the data in Figure 4 for the period where the central bank injected liquidity. We will allow $i_L^\ell = i_B > 0$ in the calibration since surplus reserves can earn a small interest rate from the central bank. We then redefine $\Psi(i_L) \equiv \psi(i_L - i_B)$ to preserve $\Psi\left(i_L^\ell\right) = 0$. The Big Four accounted for around 55% of all deposits in China in 2007 so we also set $\delta_0 = 0.55$.

To calibrate the remaining parameters, we target moments in 2014. We can then use the model to predict the 2007 values of these moments along with the credit-to-savings ratio in 2007 and 2014. The competition parameters (δ_1 and δ_2) and the WMP demand parameter (ω) are calibrated to match funding outcomes in 2014. As per Figure 2, WMPs were around 25% of GDP at the end of 2014. This is equivalent to 15% of total savings. Small banks accounted for roughly two-thirds of WMPs so we will target $W_j = 0.10$ and $W_k = 0.05$ for 2014. We will also target a funding share of $X_k = 0.45$ for 2014 since the Big Four accounted for roughly 45% of all savings (i.e., traditional deposits plus WMPs) in that year.

We allow big and small banks to have different operating cost parameters, ϕ_k and ϕ_j. China has around 200 commercial banks so a big bank is on average 40 times as large as a small bank (i.e., $\frac{0.45}{4}/\frac{0.55}{196}$). In the context of our model, this size difference implies that the big bank faces ϕ_k below ϕ_j. To match the observed size difference, we set $\phi_j = 40\phi_k$ so that marginal operating costs are the same across banks.[22] We then calibrate ϕ_k to match a loan-to-deposit ratio of 0.70 for the Big Four in 2014. We will check that the resulting operating cost parameters are high enough to deliver negligible WMP issuance in 2007.

Lastly, we calibrate the average liquidity shock, $\overline{\theta} \equiv \pi\theta_\ell + (1-\pi)\theta_h$, to get an average interbank rate of 3.6% when $\alpha = 0.25$. The 3.6% target is the weighted average seven-day interbank repo rate in 2014. The seven-day rate is the longest maturity for which there is significant trading volume. It is difficult to target shorter-term (e.g., overnight) repo rates since we are working with a two-period model and each period must be long enough to match reasonable data on loan returns (i_A). This is just a level effect though: the correlation between the overnight and seven-day repo rates is around 0.95.

The results are summarized in Table 1.[23] Our model generates most of the rise in WMPs between 2007 and 2014. It also generates half of the increase in the average seven-day

[22] Differences in ϕ can be interpreted as differences in retail networks that stem from exogenous social or political forces. In robustness checks, we found that cutting the $\frac{\phi_j}{\phi_k}$ ratio to five (based on the size difference between the Big Four and only the JSCBs) and re-calibrating the model generates very similar results.

[23] The calibrated parameters are: $\omega = 126.84$, $\delta_1 = 266.36$, $\delta_2 = 0.374$, $\phi_k = 0.0335$, and $\overline{\theta} = 0.1325$.

interbank repo rate. Since yearly averages can mask some of the most severe events, it is also useful to consider the peak interbank rates observed before and after CBRC's enforcement action (10.1% and 11.6% respectively, as measured by daily averages). Of this 150 basis point increase in peak rates, our model delivers 90 basis points. Returning to Table 1, the model also delivers more than half of the decrease in the Big Four's funding share. We also obtain a large increase in the Big Four's loan-to-deposit ratio, although the increase is somewhat larger than what we observe in the data. Finally, we obtain a sizeable 3.2 percentage point increase in the aggregate credit-to-savings ratio.

How does a 3.2 percentage point increase compare to China's overall credit boom? Commercial banks for which Bankscope has complete data collectively added RMB 40 trillion of new loans between 2007 and 2014, pushing the ratio of traditional lending to GDP from 75% in 2007 to 95% in 2014. We also know from Figure 2 that WMPs outstanding ballooned from 2% of GDP in 2007 to nearly 25% of GDP in 2014. CBRC estimates that roughly two-thirds of WMP balances in 2012 and 2013 were non-guaranteed and hence off-balance-sheet. We thus estimate that China's shadow banking system grew from a negligible fraction of GDP in 2007 to 16% of GDP in 2014. Appendix D shows that our estimate of the growth of shadow banking in China based on off-balance-sheet WMPs accounts for the majority of the growth in broader measures of shadow banking that can be constructed using data from China's National Bureau of Statistics. Adding the growth of the traditional and shadow sectors, we get a 35 percentage point increase in the ratio of total credit to GDP. This translates into a roughly 10 percentage point increase in China's credit-to-savings ratio, from 65% in 2007 to 75% in 2014. Appendix E shows that the RMB 4 trillion fiscal stimulus package undertaken in 2009 and 2010 explains around 40% of this 10 percentage point increase. Therefore, our model generates one-third of China's overall credit boom and over one-half of what is unexplained by the government's stimulus package.

5.2 External Validation

We now examine the external validity of the calibrated model by checking whether it can match empirical moments not targeted in the calibration, namely observed correlations between the interbank rate and the returns to WMPs issued by small and big banks. Table 2 reports these correlations, calculated using monthly data from January 2008 to December 2014. The time series for i_L is the average interbank repo rate weighted by transaction volume. The time series for ξ_j and ξ_k are the average returns promised by small and big banks respectively on 3-month WMPs. Since Wind has only partial data on the amount of funding raised by each WMP, ξ_j and ξ_k are unweighted averages. We will introduce error terms to absorb imperfections in the measurement of ξ_j and ξ_k.

Table 2 shows that i_L is positively correlated with each of ξ_j, ξ_k, and $\xi_j - \xi_k$. It also shows that ξ_j is positively correlated with each of ξ_k and $\xi_j - \xi_k$ while the correlation between ξ_k and $\xi_j - \xi_k$ is not significant. We would like to know the extent to which our calibrated model can replicate these correlations. To this end, we allow α, the parameter governing liquidity regulation, to be drawn from a normal distribution:

$$\alpha = \bar{\alpha} + \varepsilon_\alpha \tag{21}$$

where ε_α is normally distributed with mean 0 and variance σ_α^2. We set $\bar{\alpha} = \frac{0.14+0.25}{2} = 0.195$, where $\alpha = 0.14$ generated the initial loan-to-deposit ratio of 86% for small banks in Section 5.1 and $\alpha = 0.25$ generated the regulated ratio of 75%. We draw values of α using equation (21) and simulate the model for each value to generate the average interbank rate, $\pi i_L^\ell + (1-\pi)\, i_L^h$, the WMP returns offered by small banks, $\xi_j + \varepsilon_{\xi_j}$, and the WMP returns offered by big banks, $\xi_k + \varepsilon_{\xi_k}$. Here, ε_{ξ_j} and ε_{ξ_k} denote measurement errors which are drawn from two independent normal distributions with mean 0 and variances $\sigma_{\xi_j}^2$ and $\sigma_{\xi_k}^2$ respectively.[24] We then use Simulated Method of Moments to estimate the three unknown parameters σ_α, σ_{ξ_j}, and σ_{ξ_k}. Appendix F describes the estimation procedure in more detail.

The first column of Table 3 reports the estimated parameter values (Panel A) and predicted correlations (Panel B). The observed correlations from Table 2 appear in the last column of Panel B. Notice that σ_α is quantitatively large and highly significant. Also notice that the estimated model matches very well the observed correlations between i_L and each of ξ_j, ξ_k, and $\xi_j - \xi_k$. Shocks to α are therefore important for generating the right correlations between the interbank rate and WMP returns. At the same time though, the estimated model matches less well the pairwise correlations among WMP returns. It will thus be useful to also allow for other shocks, as is done next.

5.3 Analysis of Other Shocks

We consider two other shocks: demand shocks and money supply shocks. We repeat the exercise conducted in the first column of Table 3 for each of these shocks separately. We then consider a version of the quantitative model which has demand shocks, money supply shocks, and shocks to liquidity regulation at the same time.

[24] All distributions are truncated to avoid abnormal values of α, ξ_j, and ξ_k.

Demand shocks are introduced by allowing i_A to exceed a floor $\bar{\imath}_A$. Specifically:

$$i_A = \bar{\imath}_A + |\varepsilon_{i_A}|$$

where ε_{i_A} is normally distributed with mean 0 and variance $\sigma^2_{i_A}$. The floor represents the benchmark loan rate after the highest permissible discount is applied. Demand shocks have their own importance in China given the fiscal stimulus package discussed earlier. An exogenous increase in i_A relative to $\bar{\imath}_A$ captures the possibility that the government increased the attractiveness of lending by giving banks extra encouragement to fund the stimulus package. We simulate the model for different values of i_A while holding $\alpha = \bar{\alpha}$. The results are reported in the second column of Table 3. The estimated value of σ_{i_A} in Panel A is not significantly different from zero and the overall fit in Panel B is much worse than the model with only variations in α. Intuitively, banks will want a higher funding share when investment opportunities become more attractive. Funding shares are given by equations (10) and (11) so, all else constant, both ξ_j and ξ_k increase following an increase in i_A. However, big banks internalize that an increase in ξ_k will push small banks to increase ξ_j even further, forcing the big banks to increase ξ_k by even more in order to change their funding share. Each individual small bank takes the actions of other banks as given so there is no similar ratchet effect on ξ_j. This makes the response of ξ_k to i_A more dramatic than the response of ξ_j to i_A. As a result, the correlation between i_L and ξ_k is stronger than the correlation between i_L and ξ_j in the model with only shocks to i_A. The correlation between i_L and $\xi_j - \xi_k$ in the second column of Table 3 is then negative, contradicting the positive correlation in the data.

Money supply shocks are introduced by allowing for exogenous variation in external liquidity:

$$\Psi(i_L) = \psi(i_L - i_B) + \varepsilon_\Psi$$

where ε_Ψ is normally distributed with mean 0 and variance σ^2_Ψ. We simulate the model for different draws of ε_Ψ while holding $\alpha = \bar{\alpha}$ and $i_A = \bar{\imath}_A$. Note that i_L is always endogenously determined. The results are reported in the third column of Table 3. As was the case with only demand shocks, the estimated value of σ_Ψ in Panel A is not significantly different from zero and the overall fit in Panel B is much worse than the model with only variations in α. All else constant, a decrease in external liquidity increases i_L but reduces both ξ_j and ξ_k. Intuitively, the increase in i_L reflects the fact that the central bank is tightening the interbank market by removing liquidity, the decrease in ξ_j reflects the fact that small banks have less of a regulatory arbitrage motive when the interbank rate is high, and the decrease in ξ_k reflects the fact that big banks are competing against less aggressive products by the small banks. Money supply shocks thus generate negative correlations between the interbank rate and WMP returns, contradicting the positive correlations in the data.

We now allow for all three shocks in order to investigate the relative importance of each one. The shocks (ε_α, ε_{i_A}, and ε_Ψ) and measurement errors (ε_{ξ_j} and ε_{ξ_k}) are drawn from the relevant distributions, all of which are assumed to be independent of each other. We are able to separately identify σ_α, σ_{i_A}, and σ_Ψ since shocks to regulation, demand, and external liquidity imply different correlations between i_L, ξ_j, and ξ_k, as discussed above. The results are reported in the fourth column of Table 3. The quantitative model with three shocks matches the six empirical correlations almost perfectly. Moreover, σ_α, σ_{i_A}, and σ_Ψ are all statistically significant, indicating that all three shocks are relevant.[25] However, shocks to liquidity regulation turn out to be much more important than shocks to either demand or external liquidity. To this point, we find that variations in α explain 46% of the variance in i_L while variations in i_A and the intercept of $\Psi(\cdot)$ explain only 21% and 34% respectively. This echoes our finding in the calibration exercise that changes in liquidity regulation can explain about half of the increase in the interbank rate and one-third of the increase in the aggregate credit-to-savings ratio between 2007 and 2014.

6 Conclusion

This paper has developed a theoretical framework to study the endogenous response of the banking sector to liquidity regulation and the implications for the aggregate economy. We showed that stricter liquidity standards can generate unintended credit booms. The mechanism we uncovered is as follows. Liquidity minimums are endogenously more binding on small banks than on large ones. In response, small banks find it optimal to offer a new savings instrument and manage the funds raised by this instrument in an off-balance-sheet vehicle that is not subject to liquidity regulation. As small banks push to attract savings into off-balance-sheet instruments, they raise the interest rates on these instruments above the rates on traditional deposits and poach funding from big banks. Big banks respond to this competitive threat both by issuing their own high-return savings instruments and by tightening the interbank market for emergency liquidity against small banks. The new equilibrium is characterized by more credit as savings are reallocated across banks and lending is reallocated across markets.

Applying our framework to China, we found that a regulatory push to increase bank

[25] This can also be seen from estimated measurement errors: σ_{ξ_j} becomes statistically insignificant and the magnitude of σ_{ξ_k} is less than a quarter of the previous estimates.

liquidity and cap loan-to-deposit ratios in the late 2000s accounts for one-third of China's unprecedented credit boom between 2007 and 2014. We also found broad empirical support for the model's cross-sectional predictions and presented direct evidence that small banks responded to stricter liquidity regulation while large banks responded to the activities of small banks. Using transaction-level data, we also confirmed that large banks do indeed change interbank conditions to the detriment of their smaller competitors. A quantitative version of our model was able to match a broad set of moments and assigned a dominant role to variation in liquidity regulation relative to other shocks.

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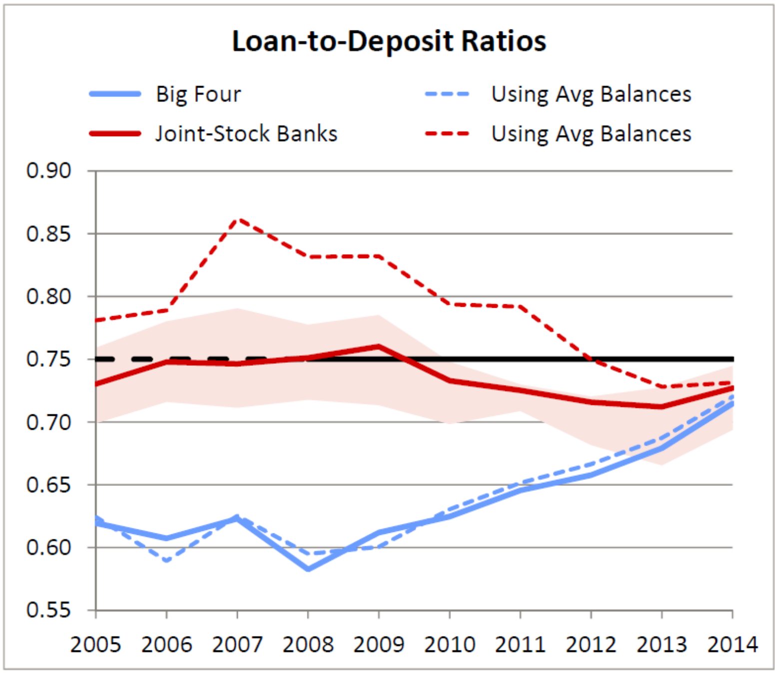
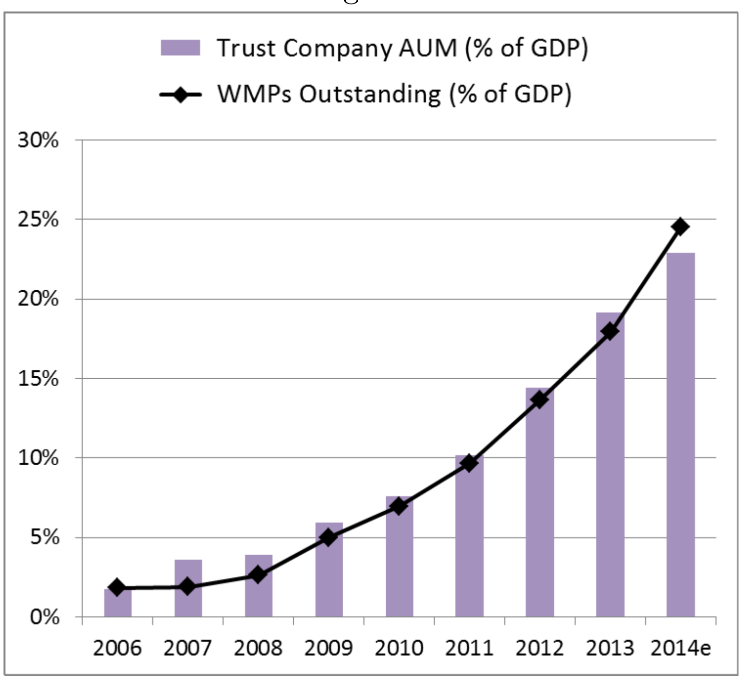
Figure 1



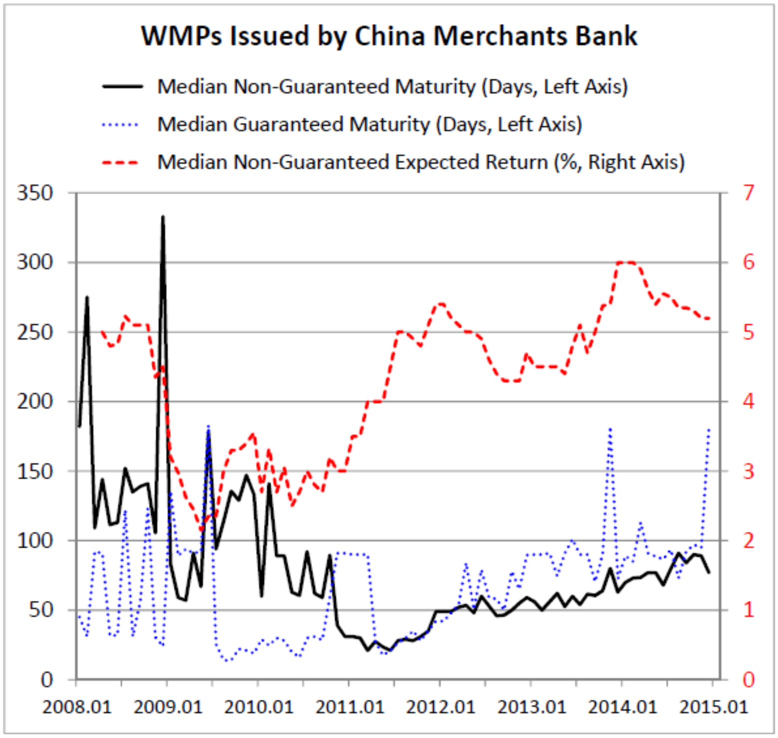
Source: Bankscope and bank annual reports. Shaded area is interquartile range.

Figure 2



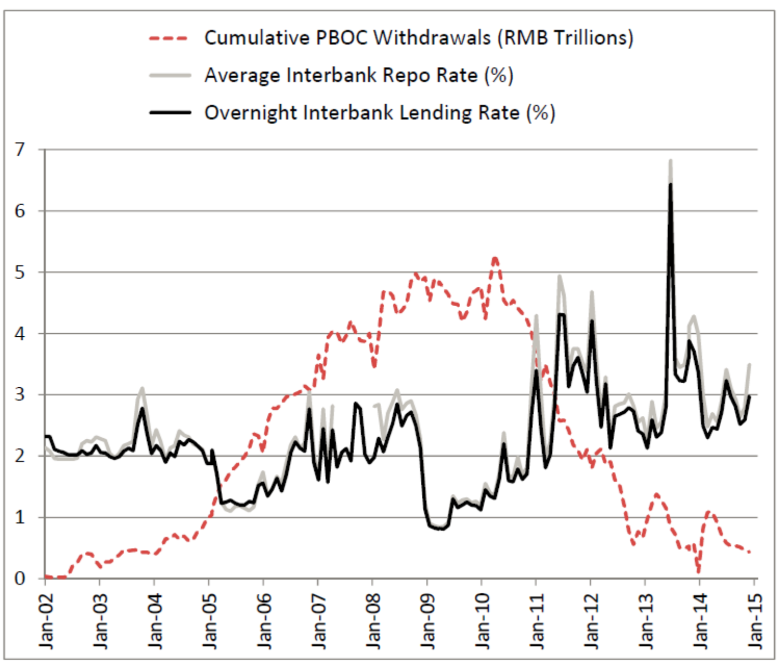
Source: PBOC, CBRC, IMF, China Trustee Association, KPMG China Trust Surveys

Figure 3



Source: Wind Financial Terminal

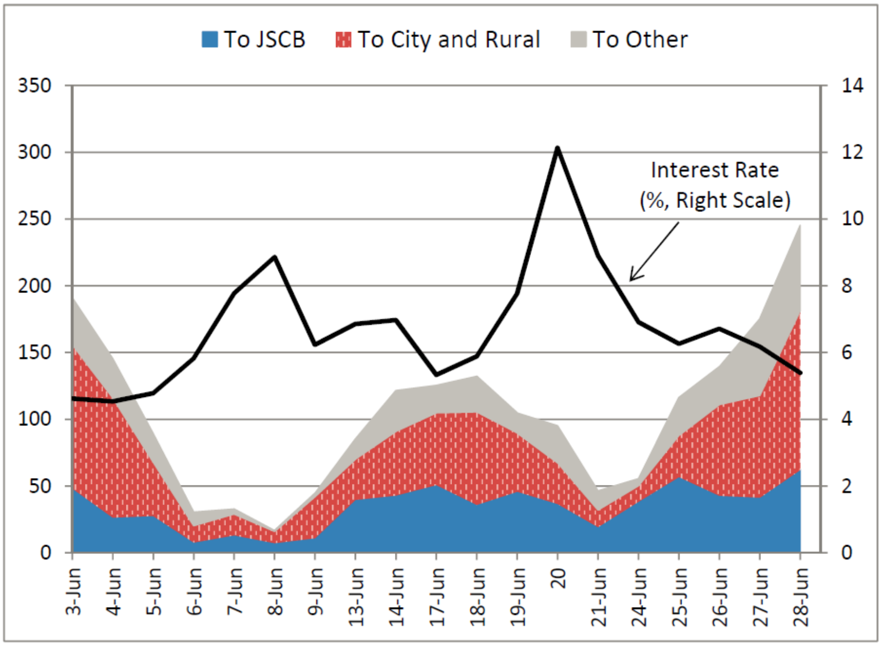
Figure 4



Source: PBOC and Wind Financial Terminal

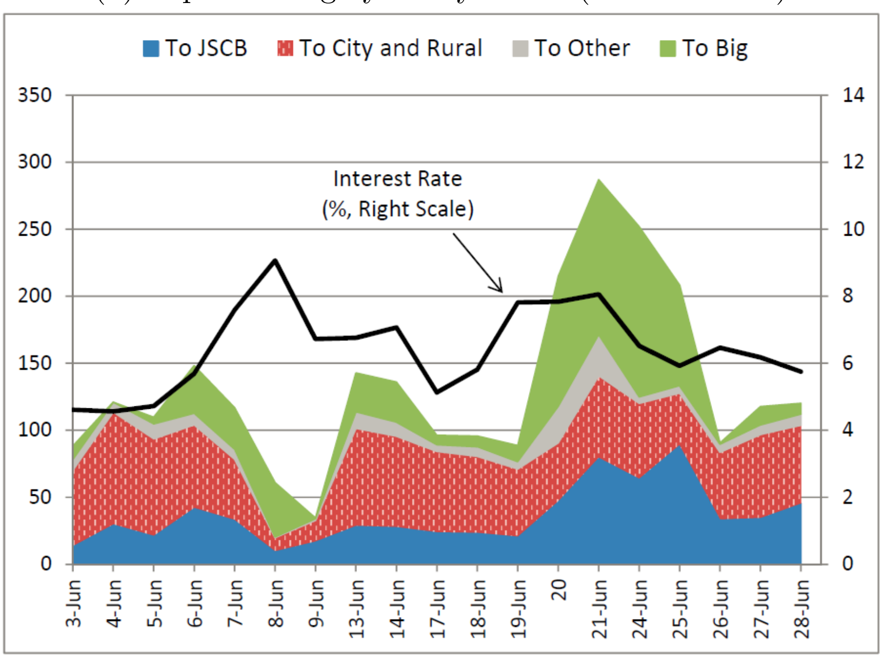
Figure 5

(a) Repo Lending by Big Banks (RMB Billions)



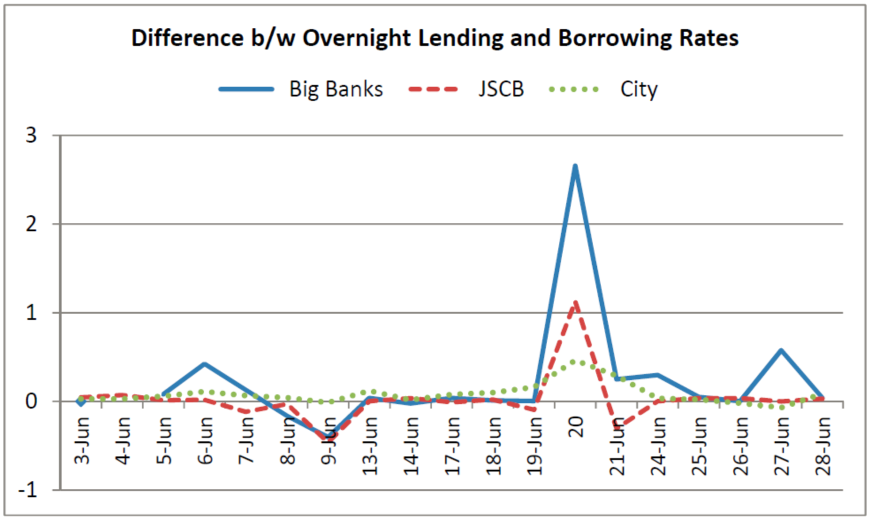
Note: Interest rate is the weighted average lending rate charged by big banks. Excludes loans between big banks.

(b) Repo Lending by Policy Banks (RMB Billions)



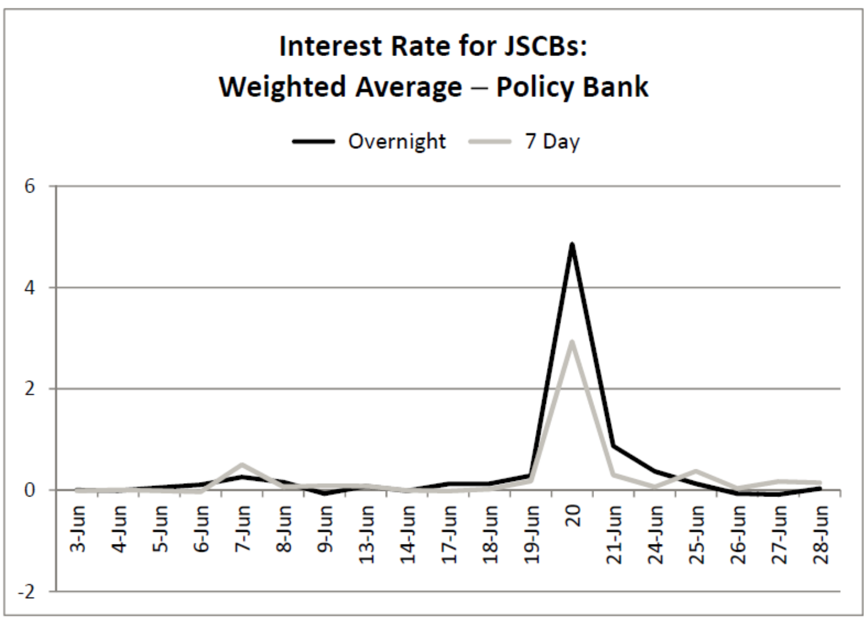
Note: Interest rate is the weighted average lending rate charged by policy banks.

Figure 6



Note: The lending (borrowing) rate is the interest rate at which the bank lends (borrows) on the interbank repo market.

Figure 7



Note: The interest rate refers to the rate on the interbank repo market.

Table 1

Calibration Results

	(1) Model $\alpha = 0.14$	(2) Data 2007	(3) Model $\alpha = 0.25$	(4) Data 2014
Average Interbank Rate $(\pi i_L^{\ell} + (1-\pi)\, i_L^h)$	3.35%	3.1%	3.6%	3.6%
Small Bank WMPs (W_j)	0.03	NA	0.10	0.10
Big Bank WMPs (W_k)	0.01	NA	0.05	0.05
Big Bank Funding Share (X_k)	0.52	0.55	0.45	0.45
Big Bank Loan-to-Deposit Ratio $(1 - \frac{R_k}{X_k})$	58%	62%	70%	70%
Credit-to-Savings Ratio $(1 - R_j - R_k)$	72.1%	65%	75.3%	75%

Notes: We target the 2014 values of all variables in this table except for the credit-to-savings ratio. The 2007 values of these variables as well as the 2007 and 2014 values of the credit-to-savings ratio are generated by the calibrated model.

Table 2

Pairwise Correlations

	i_L	ξ_j	ξ_k
ξ_j	0.456	-	-
	(0.077)		
ξ_k	0.329	0.736	-
	(0.095)	(0.052)	
$\xi_j - \xi_k$	0.259	0.550	-0.152
	(0.093)	(0.088)	(0.147)

Notes: Bootstrapped standard errors are in parentheses.

Table 3

Estimation Results

	Panel A: Parameter Values			
	Model with only σ_α	Model with only σ_{i_A}	Model with only σ_Ψ	Model with σ_α, σ_{i_A}, σ_Ψ
σ_α	0.0680	-	-	0.0281
	(3.60)			(7.40)
σ_{i_A}	-	0.0551	-	0.0004
		(1.44)		(10.40)
σ_Ψ	-	-	0.0006	0.0052
			(1.33)	(11.96)
$\sigma_{\xi_j} \times 10^4$	3.12	6.12	6.36	0.0013
	(1.91)	(1.17)	(1.76)	(0.15)
$\sigma_{\xi_k} \times 10^4$	2.48	5.68	2.38	0.4436
	(1.65)	(2.68)	(1.55)	(12.32)

	Panel B: Pairwise Correlations				
	Model with only σ_α	Model with only σ_{i_A}	Model with only σ_Ψ	Model with σ_α, σ_{i_A}, σ_Ψ	Data
$corr\left(i_L, \xi_j\right)$	0.475	0.115	-0.008	0.458	0.456
$corr\left(i_L, \xi_k\right)$	0.318	0.411	-0.002	0.331	0.329
$corr\left(i_L, \xi_j - \xi_k\right)$	0.237	-0.227	-0.006	0.263	0.259
$corr\left(\xi_j, \xi_k\right)$	0.141	0.051	-0.004	0.730	0.736
$corr\left(\xi_j, \xi_j - \xi_k\right)$	0.811	0.662	0.932	0.565	0.550
$corr\left(\xi_k, \xi_j - \xi_k\right)$	-0.465	-0.714	-0.367	-0.151	-0.152

Notes: Panel A reports the estimated parameter values. Bootstrapped t-statistics are in parentheses. Columns 1 to 4 in Panel B report the simulated correlations using the estimated parameter values in each model. Column 5 in Panel B reports the correlations in the data as per Table 2.

Appendix A – Proofs

Proof of Proposition 1

By contradiction. Suppose $\rho > 0$. If $\mu_j > 0$, then $R_j = 0$ so equation (4) implies $i_L = \frac{\overline{\theta}X}{\psi}$. Substituting into equation (9) then implies $\xi_j > 0$ if and only if $\phi < \frac{(1+i_A)^2-1}{X} - \frac{\overline{\theta}^2 X}{\psi X} \equiv \overline{\phi}_1$ (where we have used $X_j = X$ in a symmetric equilibrium). If instead $\mu_j = 0$, then equation (7) implies $i_L = (1+i_A)^2 - 1$. Substituting into equation (9) then implies $\xi_j > 0$ if and only if $\phi < \frac{1-\overline{\theta}}{X}\left[(1+i_A)^2 - 1\right] \equiv \overline{\phi}_2$. Defining $\overline{\phi} \equiv \min\left\{\overline{\phi}_1, \overline{\phi}_2\right\}$ completes the proof. ■

Proof of Proposition 2

With $\rho = 0$, the equilibrium is characterized by (4), (7), and:

$$\xi_j = \frac{\alpha \mu_j}{2\left(1-\overline{\theta}\right)}$$

$$\mu_j\left[R_j - \alpha\left(X - \omega\xi_j\right)\right] = 0 \text{ with complementary slackness}$$

There is an implicit refinement here since we are writing $\xi_j = \frac{\alpha\mu_j}{2\left(1-\overline{\theta}\right)}$ instead of $\xi_j = \frac{\alpha\mu_j\tau_j}{2\left(1-\overline{\theta}\right)}$. Both produce $\xi_j = 0$ if $\alpha\mu_j = 0$ so the refinement only applies if $\alpha\mu_j > 0$. Return to equations (8) and (9) with $\rho = 0$ and $\alpha\mu_j > 0$. If $\xi_j > 0$, then $\eta_j^1 > 0$. This implies $\tau_j = 1$ which confirms $\xi_j > 0$. If $\xi_j = 0$, then $\eta_j^1 = \eta_j^0$. This implies $\tau_j \in [0,1]$. However, any $\tau_j \in (0,1]$ would return $\xi_j > 0$, violating $\xi_j = 0$. We thus eliminate $\xi_j = 0$ by refinement. Instead, $\alpha\mu_j > 0$ is associated with $\xi_j > 0$ and thus $\tau_j = 1$. For this reason, we write $\xi_j = \frac{\alpha\mu_j}{2\left(1-\overline{\theta}\right)}$. We can now proceed with the rest of the proof. There are two cases:

1. If $\mu_j = 0$, then $\xi_j = 0$ and $1 + i_L = (1+i_A)^2$. Equation (4) then pins down R_j. To ensure that $R_j \geq \alpha\left(X - \omega\xi_j\right)$ is satisfied, we need $\alpha \leq \overline{\theta} - \frac{\psi\left[(1+i_A)^2-1\right]}{X} \equiv \widetilde{\alpha}_1$. We have now established $\xi_j = 0$ if $\alpha \leq \widetilde{\alpha}_1$.

2. If $\mu_j > 0$, then complementary slackness implies $R_j = \alpha\left(X - \omega\xi_j\right)$. Combining with the other equilibrium conditions, we find that $\mu_j > 0$ delivers:

$$i_L = \frac{\alpha^2\omega\left[(1+i_A)^2 - 1\right] - 2\left(1-\overline{\theta}\right)\left(\alpha - \overline{\theta}\right)X}{\alpha^2\omega + 2\psi\left(1-\overline{\theta}\right)} \tag{22}$$

   Verifying $\mu_j > 0$ is equivalent to verifying $1 + i_L < (1+i_A)^2$. This reduces to $\alpha > \widetilde{\alpha}_1$. If $\widetilde{\alpha}_1 \geq 0$, then we have established $\xi_j > 0$ with $\tau_j = 1$ for any $\alpha > \widetilde{\alpha}_1$.

Defining $\widetilde{\alpha} = \max\{\widetilde{\alpha}_1, 0\}$ completes the proof. ■

Proof of Proposition 3

Consider $\alpha = 0$. If $\mu_j = 0$, then (7) implies $i_L = (1+i_A)^2 - 1$ which is the highest feasible interbank rate. If instead $\mu_j > 0$, then the liquidity rule binds. In particular, $R_j = \alpha\,(X_j - \tau_j W_j)$ which is just $R_j = 0$ when $\alpha = 0$. We can then conclude $i_L = \frac{\overline{\theta}X}{\psi}$ from (4). Note that $\mu_j > 0$ is verified if and only if $\frac{\overline{\theta}X}{\psi} < (1+i_A)^2 - 1$.

Based on the results so far, we can see that the interbank rate at $\alpha = 0$ is independent of ρ. Let i_{L0} denote the interbank rate at $\alpha = 0$ and let $i_{L1}(\rho)$ denote the interbank rate at some $\alpha > 0$. From (4), we know $i_{L1}(\rho) = \frac{\overline{\theta}X}{\psi} - \frac{R_{j1}(\rho)}{\psi}$, where $R_{j1}(\rho)$ is reserve holdings at the $\alpha > 0$ being considered. The rest of the proof proceeds by contradiction. In particular, suppose $i_{L1}(\rho) > i_{L0}$. Then $\alpha = 0$ must be associated with $\mu_j > 0$, otherwise i_{L0} would be the highest feasible interbank rate and the supposition would be incorrect. We can thus write $i_{L0} = \frac{\overline{\theta}X}{\psi}$ and $i_{L1}(\rho) = i_{L0} - \frac{R_{j1}(\rho)}{\psi}$. The only way to get $i_{L1}(\rho) > i_{L0}$ is then $R_{j1}(\rho) < 0$ which is impossible. ■

Proof of Proposition 4

Start with general α. The derivatives of the big bank's objective function are:

$$\frac{\partial \Upsilon_k}{\partial \xi_k} \propto -\frac{2\omega\left(1-\overline{\theta}\right)}{1-\pi}\xi_k - \left[\frac{(1+i_A)^2-1}{1-\pi} - i_L^h\right]\frac{\partial R_k}{\partial \xi_k} + \left[\frac{(1+i_A)^2-1-\phi X_k}{1-\pi} - \theta_h i_L^h\right]\frac{\partial X_k}{\partial \xi_k}$$

$$\frac{\partial \Upsilon_k}{\partial i_L^h} \propto R_k - \theta_h X_k - \left[\frac{(1+i_A)^2-1}{1-\pi} - i_L^h\right]\frac{\partial R_k}{\partial i_L^h} + \left[\frac{(1+i_A)^2-1-\phi X_k}{1-\pi} - \theta_h i_L^h\right]\frac{\partial X_k}{\partial i_L^h}$$

It will be convenient to reduce these derivatives to a core set of variables (ξ_j, ξ_k, and i_L^h). If $\mu_j > 0$, then the complementary slackness in equation (13) implies:

$$R_j = \alpha\left(X_j - \omega\xi_j\right) \tag{23}$$

With $\delta_1 + \delta_2 = 0$ and $\overline{\xi_j} = \xi_j$, equations (10) and (11) are:

$$X_j = 1 - \delta_0 + \delta_1\left(\xi_j - \xi_k\right) \tag{24}$$

$$X_k = \delta_0 + \delta_1\left(\xi_k - \xi_j\right) \tag{25}$$

Substitute (23) to (25) into equation (12) to write:

$$R_k = \delta_0\theta_h + (1-\delta_0)\left(\overline{\theta}-\alpha\right) + \delta_1\left(\theta_h - \overline{\theta} + \alpha\right)\left(\xi_k - \xi_j\right) + \alpha\omega\xi_j - \psi i_L^h \tag{26}$$

Finally, combine equations (14) and (15) to get:

$$\xi_j = \frac{\alpha(1-\pi)}{2\left(1-\overline{\theta}\right)}\left[\frac{(1+i_A)^2-1}{1-\pi} - i_L^h\right] \tag{27}$$

We can now write $\frac{\partial\Upsilon_k}{\partial\xi_k} = 0$ as:

$$\xi_k = \frac{\delta_1\left[\left(1-\theta_h+\overline{\theta}-\alpha\right)\left[(1+i_A)^2-1\right] - \phi\delta_0 + \phi\delta_1\xi_j - \left(\overline{\theta}-\alpha\right)(1-\pi)\,i_L^h\right]}{2\omega\left(1-\overline{\theta}\right)+\phi\delta_1^2} \tag{28}$$

We can also write $\frac{\partial\Upsilon_k}{\partial i_L^h} = 0$ as:

$$\begin{aligned} i_L^h \;&=\; \frac{\left[\frac{\psi}{1-\pi} + \frac{\alpha\left[\alpha\omega+\delta_1\left(1-\theta_h+\overline{\theta}-\alpha\right)\right]}{2\left(1-\overline{\theta}\right)}\right]\left[(1+i_A)^2-1\right]}{2\psi + \frac{\alpha(1-\pi)}{2\left(1-\overline{\theta}\right)}\left[\alpha\omega+\delta_1\left(\overline{\theta}-\alpha\right)\right]} \\ &\quad + \frac{(1-\delta_0)\left(\overline{\theta}-\alpha\right) - \frac{\alpha\phi\delta_0\delta_1}{2\left(1-\overline{\theta}\right)} + \alpha\omega\xi_j - \delta_1\left[\overline{\theta}-\alpha+\frac{\alpha\phi\delta_1}{2\left(1-\overline{\theta}\right)}\right]\left(\xi_k-\xi_j\right)}{2\psi + \frac{\alpha(1-\pi)}{2\left(1-\overline{\theta}\right)}\left[\alpha\omega+\delta_1\left(\overline{\theta}-\alpha\right)\right]} \end{aligned} \tag{29}$$

Remark 1 *If the big bank's inequality constraints are non-binding, the equilibrium is a triple* $\left\{\xi_j, \xi_k, i_L^h\right\}$ *that solves (27), (28), and (29). It must then be verified that the solution to these equations satisfies* $\xi_k \geq 0$ *along with* $R_k > \alpha X_k$ *and* $\mu_j > 0$*. The big bank is technically indifferent between any* $\tau_k \in [0,1]$ *if its liquidity rule is slack so, for analytical convenience, consider* $\tau_k = 0$*. We also need to check* $W_j \leq X_j$ *and* $W_k \leq X_k$ *so that deposits are non-negative. Finally, we want to check that* $i_L^\ell = 0$ *does not result in a liquidity shortage when the big bank realizes* θ_ℓ *at* $t = 1$*.*

The rest of this proof focuses on $\alpha = 0$. Notice $\xi_j = 0$ from (27). As discussed in the main text, we also want $\xi_k = 0$. Subbing $\alpha = 0$ and $\xi_j = \xi_k = 0$ into (28) and (29) yields:

$$\delta_1\left[\frac{\left(1-\theta_h+\overline{\theta}\right)\left[(1+i_A)^2-1\right]-\phi\delta_0}{\overline{\theta}(1-\pi)} - i_L^h\right] = 0 \tag{30}$$

$$i_L^h = \frac{(1+i_A)^2-1}{2(1-\pi)} + \frac{\overline{\theta}(1-\delta_0)}{2\psi} \tag{31}$$

To verify $\xi_k = 0$, we must verify that (30) holds when i_L^h is given by (31). This requires either $\delta_1 = 0$ or:

$$\phi = \frac{1}{\delta_0}\left[1 - \theta_h + \frac{\overline{\theta}}{2}\right]\left[(1 + i_A)^2 - 1\right] - \frac{\overline{\theta}^2 (1 - \pi)(1 - \delta_0)}{2\psi\delta_0} \equiv \phi^* \tag{32}$$

In other words, we can use either $\delta_1 = 0$ or the combination of $\delta_1 > 0$ and $\phi = \phi^*$ to get ξ_k exactly zero at $\alpha = 0$. Note that $W_j \leq X_j$ and $W_k \leq X_k$ are trivially true with $\xi_j = \xi_k = 0$. We now need to check $R_k > \alpha X_k$ and $\mu_j > 0$. Using (14) and (31), rewrite $\mu_j > 0$ as:

$$\frac{(1 + i_A)^2 - 1}{1 - \pi} > \frac{\overline{\theta}(1 - \delta_0)}{\psi} \tag{33}$$

Note that condition (33) is also sufficient for $\phi^* > 0$. With $\mu_j > 0$ verified, we can substitute $\alpha = 0$ into equation (23) to get $R_j = 0$. The next step is to check $R_k > \alpha X_k$ which is simply $R_k > 0$ at $\alpha = 0$. Recall that R_k is given by equation (26). Use $\alpha = 0$ and $\xi_j = \xi_k = 0$ along with i_L^h as per (31) to rewrite equation (26) as:

$$R_k = \theta_h\delta_0 + \frac{\overline{\theta}(1 - \delta_0)}{2} - \psi\frac{(1 + i_A)^2 - 1}{2(1 - \pi)} \tag{34}$$

The condition for $R_k > 0$ is therefore:

$$\frac{(1 + i_A)^2 - 1}{1 - \pi} < \frac{\overline{\theta}(1 - \delta_0)}{\psi} + \frac{2\delta_0\theta_h}{\psi} \tag{35}$$

The last step is to check that there is sufficient liquidity at $t = 1$ when the big bank's liquidity shock is low. The demand for liquidity in this case will be $\overline{\theta}X_j + \theta_\ell X_k$. The supply of liquidity will be $R_j + R_k$ since we have fixed $i_L^\ell = 0$. We already know $\xi_j = \xi_k = 0$ at $\alpha = 0$. Therefore, $X_j = 1 - \delta_0$ and $X_k = \delta_0$. We also know $R_j = 0$ and R_k as per (34). Therefore, $R_j + R_k \geq \overline{\theta}X_j + \theta_\ell X_k$ can be rewritten as:

$$\frac{(1 + i_A)^2 - 1}{1 - \pi} \leq \frac{2\delta_0(\theta_h - \theta_\ell)}{\psi} - \frac{\overline{\theta}(1 - \delta_0)}{\psi} \tag{36}$$

Condition (36) is stricter than (35) so we can drop (35). We now just need to make sure that conditions (33) and (36) are not mutually exclusive. Using $\overline{\theta} \equiv \pi\theta_\ell + (1 - \pi)\theta_h$, this requires:

$$\theta_\ell < \left[1 - \frac{1 - \delta_0}{\delta_0 + \pi(1 - \delta_0)}\right]\theta_h \tag{37}$$

The right-hand side of (37) is positive if and only if:

$$\pi > \frac{1-2\delta_0}{1-\delta_0} \tag{38}$$

Therefore, with θ_ℓ sufficiently low and π sufficiently high, conditions (33) and (36) define a non-empty interval for i_A, completing the proof. ■

Proof of Proposition 5

Fixed Funding Share Impose $\alpha = \overline{\theta}$ and $\delta_1 = 0$ on equations (27), (28), and (29). The resulting system can be written as $\xi_k = 0$ and:

$$\xi_j = \frac{\frac{\overline{\theta}\psi}{2}\left[(1+i_A)^2 - 1\right]}{2\psi\left(1-\overline{\theta}\right) + \omega\overline{\theta}^2(1-\pi)} \tag{39}$$

$$i_L^h = \frac{\left[\frac{\psi\left(1-\overline{\theta}\right)}{1-\pi} + \omega\overline{\theta}^2\right]\left[(1+i_A)^2 - 1\right]}{2\psi\left(1-\overline{\theta}\right) + \omega\overline{\theta}^2(1-\pi)} \tag{40}$$

With $\delta_1 = 0$ in equations (24) and (25), the funding shares are $X_j = 1-\delta_0$ and $X_k = \delta_0$. Impose along with $\alpha = \overline{\theta}$ on equations (23) and (26) to get:

$$R_k = \theta_h\delta_0 + \omega\overline{\theta}\xi_j - \psi i_L^h$$

$$R_j + R_k = \overline{\theta}(1-\delta_0) + \theta_h\delta_0 - \psi i_L^h$$

where ξ_j and i_L^h are given by (39) and (40) respectively. We now need to go through all the steps in Remark 1 to establish the equilibrium for $\alpha = \overline{\theta}$ and fixed funding shares. Using equations (14) and (40), we can see that $\mu_j > 0$ is trivially true. Using $\xi_k = 0$ and $X_k = \delta_0$, we can also see that $W_k \leq X_k$ is trivially true. The condition for $W_j \leq X_j$ is:

$$\frac{(1+i_A)^2 - 1}{1-\pi} \leq \frac{2(1-\delta_0)}{\psi}\left[\overline{\theta} + \frac{2\psi\left(1-\overline{\theta}\right)}{\omega\overline{\theta}(1-\pi)}\right] \tag{41}$$

The conditions for $R_k > \overline{\theta}X_k$ and $R_j + R_k \geq \overline{\theta}X_j + \theta_\ell X_k$ are respectively:

$$\frac{(1+i_A)^2 - 1}{1-\pi} < \frac{2\pi(\theta_h - \theta_\ell)\delta_0}{\psi} \tag{42}$$

$$\frac{(1+i_A)^2-1}{1-\pi} \leq \frac{2\psi\left(1-\overline{\theta}\right)+\omega\overline{\theta}^2(1-\pi)}{\psi\left(1-\overline{\theta}\right)+\omega\overline{\theta}^2(1-\pi)} \frac{(\theta_h-\theta_\ell)\,\delta_0}{\psi} \tag{43}$$

Now, for the interbank rate to increase when moving from $\alpha = 0$ to $\alpha = \overline{\theta}$, we need (40) to exceed (31). Equivalently, we need:

$$\frac{(1+i_A)^2-1}{1-\pi} > \frac{\overline{\theta}(1-\delta_0)}{\psi}\left[1+\frac{2\psi\left(1-\overline{\theta}\right)}{\omega\overline{\theta}^2(1-\pi)}\right] \tag{44}$$

We must now collect all the conditions involved in the $\alpha = 0$ and $\alpha = \overline{\theta}$ equilibria and make sure they are mutually consistent. There are two lowerbounds on i_A, namely (33) and (44). Condition (44) is clearly stricter so it is the relevant lowerbound. There are also four upperbounds on i_A, namely (36), (41), (42), and (43). For the lowerbound in (44) to not violate any of these upperbounds, we need:

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega(1-\pi)} < \overline{\theta}^2 \min\left\{\frac{\pi(\theta_h-\theta_\ell)\,\delta_0}{\overline{\theta}(1-\delta_0)} - \frac{1}{2}, \frac{(\theta_h-\theta_\ell)\,\delta_0}{\overline{\theta}(1-\delta_0)} - 1\right\}$$

This inequality is only possible if the right-hand side is positive. Therefore, we need:

$$\theta_\ell < \left[1 - \frac{1-\delta_0}{\min\left\{\delta_0+\pi(1-\delta_0),\pi(1+\delta_0)\right\}}\right]\theta_h \tag{45}$$

Once again, the right-hand side must be positive so we need:

$$\pi > \max\left\{\frac{1-2\delta_0}{1-\delta_0}, \frac{1-\delta_0}{1+\delta_0}\right\} \tag{46}$$

Notice that (45) and (46) are just refinements of (37) and (38). We can now conclude that the model with fixed funding shares generates the desired results under the following conditions: π sufficiently high, θ_ℓ and $\frac{\psi}{\omega}$ sufficiently low, and i_A within an intermediate range. $\square$

Endogenous Funding Share Return to equations (27), (28), and (29). Impose $\alpha = \overline{\theta}$ and $\delta_1 = \omega$ with $\phi = \phi^*$ as per (32). Combine to get:

$$i_L^h = \frac{(1+i_A)^2-1}{1-\pi} - \frac{\left[\frac{2\psi}{1-\pi}+\frac{\omega\overline{\theta}^2}{2\left(1-\overline{\theta}\right)+\phi^*\omega}\right]\frac{(1+i_A)^2-1}{2(1-\pi)} - \frac{\omega\overline{\theta}^3(1-\delta_0)}{2\psi\left[2\left(1-\overline{\theta}\right)+\phi^*\omega\right]}}{\frac{2\psi}{1-\pi}+\frac{\omega\overline{\theta}^2}{2\left(1-\overline{\theta}\right)}\left[2+\frac{\phi^*\omega}{2\left(1-\overline{\theta}\right)+\phi^*\omega}\right]} \tag{47}$$

$$\xi_k = \frac{\frac{\overline{\theta}(1-\pi)}{2}\left[\frac{\overline{\theta}(1-\delta_0)}{\psi} + \left(\frac{\phi^*\omega}{1-\overline{\theta}} - 1\right)\frac{(1+i_A)^2-1}{1-\pi} - \frac{\phi^*\omega}{1-\overline{\theta}} i_L^h\right]}{2\left(1-\overline{\theta}\right) + \phi^*\omega} \tag{48}$$

We now need to go through the steps in Remark 1 to establish the equilibrium for $\alpha = \overline{\theta}$ and endogenous funding shares. The expressions here are more complicated so we proceed by finding one value of i_A that satisfies all the steps in Remark 1. A continuity argument will then allow us to conclude that all the steps are satisfied for a non-empty range of i_A.

Consider i_A such that:

$$\frac{(1+i_A)^2 - 1}{1-\pi} = \frac{\overline{\theta}}{\psi} \tag{49}$$

Substituting into (32) then pins down ϕ^* as:

$$\phi^* = \frac{\overline{\theta}\,(1-\pi)}{\psi}\left[\frac{1-\theta_h}{\delta_0} + \frac{\overline{\theta}}{2}\right] \tag{50}$$

From the proof of Proposition 4, we already have (33) and (36) as restrictions on i_A. We also have (37) as an upperbound on θ_ℓ and (38) as a lowerbound on π. It is easy to see that i_A as defined in (49) satisfies (33). For (49) to also satisfy (36), we need:

$$\theta_\ell < \left[1 - \frac{2-\delta_0}{2\delta_0 + \pi\,(2-\delta_0)}\right]\theta_h \tag{51}$$

$$\pi > \frac{2-3\delta_0}{2-\delta_0} \tag{52}$$

Conditions (51) and (52) are stricter than (37) and (38). We can thus drop (37) and (38).

The first step is to verify $\mu_j > 0$. Use (14) and (47) to write $\mu_j > 0$ as:

$$\frac{(1+i_A)^2 - 1}{1-\pi}\left[1 + \frac{2\psi\left[2\left(1-\overline{\theta}\right) + \phi^*\omega\right]}{\omega\overline{\theta}^2\,(1-\pi)}\right] > \frac{\overline{\theta}\,(1-\delta_0)}{\psi}$$

This is true by condition (33).

The second step is to verify $\xi_k > 0$. Substituting (47) into (48), we see that we need:

$$\frac{(1+i_A)^2 - 1}{1-\pi}\left[1 - \frac{\phi^*}{\frac{2\left(1-\overline{\theta}\right)}{\omega} + \frac{\overline{\theta}^2(1-\pi)}{\psi}}\right] < \frac{\overline{\theta}\,(1-\delta_0)}{\psi} \tag{53}$$

Using i_A as per (49) and ϕ^* as per (50):

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)} < \frac{\overline{\theta}}{2\delta_0^2}\underbrace{\left[1-\theta_h-\overline{\theta}\delta_0\left(\delta_0-\frac{1}{2}\right)\right]}_{\text{call this } Z_1} \tag{54}$$

If $Z_1 > 0$, then (54) requires $\frac{\psi}{\omega}$ sufficiently low. Note that $Z_1 > 0$ can be made true for any $\delta_0 \in (0,1)$ by assuming $\overline{\theta} < 2\left(1-\theta_h\right)$ or, equivalently, $\theta_\ell < \frac{2-(3-\pi)\theta_h}{\pi}$. This is another positive ceiling on θ_ℓ provided $\pi > 3 - \frac{2}{\theta_h}$.

The third step is to verify $R_k > \overline{\theta}X_k$. Use $\alpha = \overline{\theta}$ and $\delta_1 = \omega$ to rewrite (25) and (26) as:

$$X_k = \delta_0 + \omega\left(\xi_k - \xi_j\right) \tag{55}$$

$$R_k = \delta_0\theta_h + \omega\theta_h\xi_k - \omega\left(\theta_h - \overline{\theta}\right)\xi_j - \psi i_L^h \tag{56}$$

Therefore, $R_k > \overline{\theta}X_k$ requires:

$$i_L^h < \frac{\delta_0\left(\theta_h-\overline{\theta}\right)}{\psi} + \frac{\omega\left(\theta_h-\overline{\theta}\right)}{\psi}\left(\xi_k-\xi_j\right) + \frac{\omega\overline{\theta}}{\psi}\xi_j$$

Use (48) to replace ξ_k and (27) with $\alpha = \overline{\theta}$ to replace ξ_j:

$$\begin{aligned}
&\left[1+\frac{\omega\overline{\theta}\left(1-\pi\right)}{2\psi\left(1-\overline{\theta}\right)}\left[\overline{\theta}-\frac{2\left(1-\overline{\theta}\right)\left(\theta_h-\overline{\theta}\right)}{2\left(1-\overline{\theta}\right)+\phi^*\omega}\right]\right] i_L^h \\
< \quad & \frac{\theta_h-\overline{\theta}}{\psi}\left[\delta_0+\frac{\omega\overline{\theta}^2\left(1-\pi\right)\left(1-\delta_0\right)}{2\psi\left[2\left(1-\overline{\theta}\right)+\phi^*\omega\right]}\right] - \frac{\omega\overline{\theta}\left[\left(1+i_A\right)^2-1\right]}{2\psi}\left[\frac{3\left(\theta_h-\overline{\theta}\right)}{2\left(1-\overline{\theta}\right)+\phi^*\omega}-\frac{\overline{\theta}}{1-\overline{\theta}}\right]
\end{aligned}$$

Now use (47) to replace i_L^h and rearrange to isolate i_A:

$$\begin{aligned}
&\frac{\left(1+i_A\right)^2-1}{1-\pi}\left[2\theta_h-\frac{3\overline{\theta}}{2}+\frac{2\psi\left(1-\overline{\theta}\right)}{\omega\overline{\theta}\left(1-\pi\right)}+\frac{\omega\overline{\theta}^2\left(1-\pi\right)}{4\psi\left(1-\overline{\theta}\right)}\left(3\theta_h-4\overline{\theta}\right)+\frac{\phi^*}{\overline{\theta}}\left[\frac{\omega\overline{\theta}^2}{1-\overline{\theta}}+\frac{\psi}{1-\pi}\right]\right] \\
< \quad & \left[\frac{2\delta_0\left(\theta_h-\overline{\theta}\right)}{1-\pi}\frac{2\left(1-\overline{\theta}\right)+\phi^*\omega}{\omega\overline{\theta}}-\frac{\overline{\theta}^2\left(1-\delta_0\right)}{2\psi}\right]\left[1+\frac{\overline{\theta}^2}{2}\frac{\omega\left(1-\pi\right)}{\psi\left(1-\overline{\theta}\right)}\right] \\
& + \left(\theta_h-\overline{\theta}\right)\frac{\overline{\theta}}{\psi}\left[\frac{\omega\phi^*\delta_0}{2\left(1-\overline{\theta}\right)}+\left(1-\delta_0\right)\left[1+\frac{3\overline{\theta}^2}{4}\frac{\omega\left(1-\pi\right)}{\psi\left(1-\overline{\theta}\right)}\right]\right]
\end{aligned}$$

We can simplify a bit further by using (32) to replace all instances of $\phi^*\delta_0$ then grouping like terms:

$$
\begin{aligned}
&\frac{(1+i_A)^2-1}{1-\pi}\left[\begin{array}{l}\theta_h-\frac{\overline{\theta}}{2}+\frac{2\psi\left(1-\overline{\theta}\right)}{\omega\overline{\theta}(1-\pi)}-\frac{\omega\overline{\theta}^3(1-\pi)}{4\psi\left(1-\overline{\theta}\right)}+\frac{\phi^*}{\overline{\theta}}\left[\frac{\omega\overline{\theta}^2}{1-\overline{\theta}}+\frac{\psi}{1-\pi}\right]\\ -\left(\theta_h-\overline{\theta}\right)(1-\theta_h)\left[\frac{2}{\overline{\theta}}+\frac{3\overline{\theta}}{2}\frac{\omega(1-\pi)}{\psi\left(1-\overline{\theta}\right)}\right]\end{array}\right]\\
<\ &\left[\frac{4\delta_0\left(1-\overline{\theta}\right)\left(\theta_h-\overline{\theta}\right)}{\omega\overline{\theta}\,(1-\pi)}-\frac{\overline{\theta}^2\,(1-\delta_0)}{2\psi}\right]\left[1+\frac{\overline{\theta}^2}{2}\frac{\omega\,(1-\pi)}{\psi\left(1-\overline{\theta}\right)}\right]
\end{aligned}
$$

Substitute i_A as per (49) and ϕ^* as per (50) then rearrange:

$$
\begin{aligned}
&\frac{\psi\left(1-\overline{\theta}\right)}{\omega\,(1-\pi)}\left[\theta_h-\frac{\overline{\theta}\,(1+\delta_0)}{2}+\left[\overline{\theta}+\frac{1-\theta_h}{\delta_0}+\frac{2}{\overline{\theta}}\frac{\psi\left(1-\overline{\theta}\right)}{\omega\,(1-\pi)}\right]\left[1-\frac{2\delta_0\left(\theta_h-\overline{\theta}\right)}{\overline{\theta}}\right]\right] \qquad (57)\\
<\ &\underbrace{\frac{\overline{\theta}}{\delta_0}\left[\frac{\overline{\theta}^2\delta_0^2}{4}+\frac{\delta_0}{2}\left[3\left(\theta_h-\overline{\theta}\right)(1-\theta_h)-\overline{\theta}^2\right]-\overline{\theta}\,(1-\theta_h)\right]}_{\text{call this } Z_2}
\end{aligned}
$$

Condition (57) will be true for $\frac{\psi}{\omega}$ sufficiently low if $Z_2>0$. Use $\overline{\theta}\equiv\pi\theta_\ell+(1-\pi)\,\theta_h$ to rewrite $Z_2>0$ as:

$$
\pi^2\,(\theta_h-\theta_\ell)^2-2\left[\theta_h+\frac{(2+3\delta_0)\,(1-\theta_h)}{\delta_0\,(2-\delta_0)}\right]\pi\,(\theta_h-\theta_\ell)+\theta_h\left[\theta_h+\frac{4\,(1-\theta_h)}{\delta_0\,(2-\delta_0)}\right]<0
$$

Based on the roots of this quadratic, we can conclude that $Z_2>0$ requires:

$$
\pi\,(\theta_h-\theta_\ell)>\theta_h+\frac{(2+3\delta_0)\,(1-\theta_h)}{\delta_0\,(2-\delta_0)}-\sqrt{\frac{1-\theta_h}{2-\delta_0}\left(6\theta_h+\frac{(2+3\delta_0)^2\,(1-\theta_h)}{\delta_0^2\,(2-\delta_0)}\right)} \qquad (58)
$$

Condition (58) is satisfied by $\theta_\ell=0$ and $\pi=1$. The left-hand side is decreasing in θ_ℓ and increasing in π so it follows that $Z_2>0$ requires θ_ℓ sufficiently low and π sufficiently high.

The fourth step is to verify $W_j\le X_j$. Use $W_j=\omega\xi_j$ and (24) with $\delta_1=\omega$ to rewrite $W_j\le X_j$ as:

$$
\xi_k\le\frac{1-\delta_0}{\omega}
$$

Now use (48) with i_L^h as per (47) to replace ξ_k. Substitute i_A as per (49) and ϕ^* as per (50). Rearrange to isolate all terms with $\frac{\psi\left(1-\overline{\theta}\right)}{\omega(1-\pi)}$ on one side. The condition for $W_j\le X_j$ becomes:

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\left[\frac{\overline{\theta}^{2}}{2}+\left(1-\delta_{0}\right)\left[\overline{\theta}^{2}+\frac{\overline{\theta}\left(1-\theta_{h}\right)}{\delta_{0}}+2\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\right]\right] \tag{59}$$
$$\geq \frac{\overline{\theta}^{3}}{4}\underbrace{\left[\left(1-\theta_{h}\right)\left(3-\frac{2}{\delta_{0}}\right)-\overline{\theta}\left(1-\frac{\delta_{0}}{2}\right)\right]}_{\text{call this } Z_3}$$

A sufficient condition for $Z_3 < 0$, and hence $W_j \leq X_j$, is $\delta_0 \leq \frac{2}{3}$.

The fifth step is to verify $W_k \leq X_k$. Use $W_k = \omega\xi_k$ and (55) to rewrite $W_k \leq X_k$ as:

$$\xi_j \leq \frac{\delta_0}{\omega}$$

Now use (27) with $\alpha = \overline{\theta}$ and i_L^h as per (47) to replace ξ_j. Substitute i_A as per (49) and ϕ^* as per (50). Rearrange to isolate all terms with $\frac{\psi\left(1-\overline{\theta}\right)}{\omega(1-\pi)}$ on one side. The condition for $W_k \leq X_k$ becomes:

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\left[1-3\delta_{0}-\frac{2\delta_{0}}{\overline{\theta}}\left[\frac{1-\theta_{h}}{\delta_{0}}+\frac{2}{\overline{\overline{\theta}}}\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\right]\right] \tag{60}$$
$$\leq \frac{\overline{\theta}}{2}\underbrace{\left[\left(1-\theta_{h}\right)\left(3-\frac{1}{\delta_{0}}\right)-\overline{\theta}\left(\frac{1}{2}-\delta_{0}\right)\right]}_{\text{call this } Z_4}$$

Condition (60) will be true for $\frac{\psi}{\omega}$ sufficiently low if $Z_4 > 0$. Use the definition of $\overline{\theta}$ to rewrite $Z_4 > 0$ as:

$$\pi\left(\theta_h-\theta_\ell\right)\delta_0\left(1-2\delta_0\right) > \theta_h\delta_0\left(1-2\delta_0\right)-2\left(1-\theta_h\right)\left(3\delta_0-1\right) \tag{61}$$

If $\delta_0 \geq \frac{1}{2}$, then (61) is always true. If $\delta_0 < \frac{1}{2}$, then (61) reduces to:

$$\theta_\ell < \frac{1}{\pi}\left[\frac{2\left(1-\theta_h\right)\left(3\delta_0-1\right)}{\delta_0\left(1-2\delta_0\right)}-\theta_h\left(1-\pi\right)\right]$$

This is a positive ceiling on θ_ℓ provided $\pi > 1-\frac{2(1-\theta_h)(3\delta_0-1)}{\theta_h\delta_0(1-2\delta_0)}$ with $\delta_0 > \frac{1}{3}$. Therefore, (61) is guaranteed by θ_ℓ sufficiently low, π sufficiently high, and $\delta_0 > \frac{1}{3}$.

The sixth step is to verify feasibility of $i_L^\ell = 0$. This requires $R_j + R_k \geq \overline{\theta}X_j + \theta_\ell X_k$. Use (23) with $\alpha = \overline{\theta}$ to replace R_j. The desired inequality becomes:

$$R_k \geq \theta_\ell X_k + \omega\overline{\theta}\xi_j$$

Substituting X_k and R_k as per equations (55) and (56):

$$i_L^h \leq \frac{\theta_h - \theta_\ell}{\psi}\left[\delta_0 + \omega\left(\xi_k - \xi_j\right)\right]$$

Use (48) to replace ξ_k. Also use (27) with $\alpha = \overline{\theta}$ to replace ξ_j. Rearrange to isolate i_L^h then use (47) to replace i_L^h. Substitute i_A as per (49) and ϕ^* as per (50). Rearrange to isolate all terms with $\frac{\psi(1-\overline{\theta})}{\omega(1-\pi)}$ on one side. The feasibility condition for $i_L^\ell = 0$ becomes:

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\left[\begin{array}{c}\frac{\overline{\theta}(5-\delta_0)}{4} - \left(\theta_h - \theta_\ell\right)\left[\frac{1-\theta_h}{\overline{\theta}} + \frac{2\delta_0 - 1}{2}\right] \\ + \frac{1-\theta_h}{2\delta_0} + \left[1 - \frac{2(\theta_h - \theta_\ell)\delta_0}{\overline{\theta}}\right]\frac{1}{\overline{\overline{\theta}}}\frac{\psi\left(1-\overline{\theta}\right)}{\omega(1-\pi)}\end{array}\right] \leq \frac{3\overline{\theta}}{4}\underbrace{\left[\left(1-\theta_h\right)\left[\theta_h - \theta_\ell - \frac{\overline{\theta}}{\delta_0}\right] - \frac{\overline{\theta}^2}{2}\right]}_{\text{call this } Z_5} \quad (62)$$

Condition (62) will be true for $\frac{\psi}{\omega}$ sufficiently low if $Z_5 > 0$. Use the definition of $\overline{\theta}$ to rewrite $Z_5 > 0$ as:

$$\pi^2\left(\theta_h - \theta_\ell\right)^2 - 2\left[\pi\theta_h + \frac{\left(\pi + \delta_0\right)\left(1-\theta_h\right)}{\delta_0}\right]\left(\theta_h - \theta_\ell\right) + \theta_h\left[\theta_h + \frac{2\left(1-\theta_h\right)}{\delta_0}\right] < 0$$

Based on the roots of this quadratic, we can conclude that $Z_5 > 0$ requires:

$$\theta_\ell < \frac{1}{\pi^2}\left[\sqrt{2\pi\theta_h\left(1-\theta_h\right) + \frac{\left(\pi+\delta_0\right)^2\left(1-\theta_h\right)^2}{\delta_0^2}} - \frac{\left(\pi+\delta_0\right)\left(1-\theta_h\right)}{\delta_0} - \theta_h\pi\left(1-\pi\right)\right]$$

This is a positive upperbound on θ_ℓ provided $\frac{\theta_h(1-\pi)^2}{2(1-\theta_h)} + \frac{1-\pi}{\delta_0} < 1$. Therefore, $Z_5 > 0$ requires θ_ℓ sufficiently low and π sufficiently high.

It now remains to check that the interbank rate increases when moving from $\alpha = 0$ to $\alpha = \overline{\theta}$. This requires (47) to exceed (31) or, equivalently:

$$\frac{\left(1+i_A\right)^2 - 1}{1-\pi} > \left(1-\delta_0\right)\left[\frac{\overline{\theta}}{\psi} + \frac{4\left(1-\overline{\theta}\right)}{\omega\overline{\theta}\left(1-\pi\right)}\frac{2\left(1-\overline{\theta}\right) + \phi^*\omega}{2\left(1-\overline{\theta}\right) + 3\phi^*\omega}\right]$$

Using i_A as per (49) and ϕ^* as per (50):

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\left[\frac{1-\theta_h}{\delta_0}+\frac{\overline{\theta}\left(1-2\delta_0\right)}{2\left(1-\delta_0\right)}+\frac{2}{\overline{\theta}}\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\right]<\frac{3\overline{\theta}^2}{4\left(1-\delta_0\right)}\left[1-\theta_h+\frac{\overline{\theta}\delta_0}{2}\right] \tag{63}$$

The right-hand side is positive so (63) will be true for $\frac{\psi}{\omega}$ sufficiently low.

Putting everything together, we have shown that the model with endogenous funding shares generates the desired results under the following conditions: π sufficiently high, θ_ℓ and $\frac{\psi}{\omega}$ sufficiently low, $\delta_0 \in \left(\frac{1}{3},\frac{2}{3}\right)$, and i_A as per (49). The results then extend to a non-empty range of i_A by continuity. □

Comparison We now compare the interbank rate increases in the fixed share and endogenous share models. Notice from the proof of Proposition 4 that the interbank rate at $\alpha=0$ is the same in both models. Therefore, we just need to show that the interbank rate in the endogenous share model exceeds the interbank rate in the fixed share model at $\alpha=\overline{\theta}$. In other words, we need to show that (47) exceeds (40) for a given set of parameters. This reduces to:

$$\frac{\left(1+i_A\right)^2-1}{1-\pi}\left[1-\frac{\phi^*}{\frac{2\left(1-\overline{\theta}\right)}{\omega}+\frac{\overline{\theta}^2\left(1-\pi\right)}{\psi}}\right]<\frac{\overline{\theta}\left(1-\delta_0\right)}{\psi}$$

which is exactly (53), where (53) was the condition for $\xi_k>0$ at $\alpha=\overline{\theta}$ in the endogenous share model. To complete the proof, we must now show that there are indeed parameters that satisfy the conditions in both models. For $\alpha=0$, we imposed conditions (33) and (36) along with π sufficiently high and θ_ℓ sufficiently low. These conditions applied to both models. For $\alpha=\overline{\theta}$ in the fixed share model, we also imposed conditions (41), (42), (43), and (44) along with $\frac{\psi}{\omega}$ sufficiently low. For $\alpha=\overline{\theta}$ in the endogenous share model, we added $\delta_0\in\left(\frac{1}{3},\frac{2}{3}\right)$ and i_A in the neighborhood of (49). In (51) and (52), we showed that π sufficiently high and θ_ℓ sufficiently low make (49) satisfy condition (36). We have also shown that condition (44) is stricter than condition (33). Therefore, we just need to show that (49) satisfies conditions (41), (42), (43), and (44). Substituting i_A as per (49) into these conditions produces the following inequalities which we must check:

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}>\frac{\overline{\theta}^2\left(2\delta_0-1\right)}{4\left(1-\delta_0\right)} \tag{64}$$

$$\theta_\ell<\left[1-\frac{1}{\pi\left(1+2\delta_0\right)}\right]\theta_h \tag{65}$$

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}\left[1-\frac{2\left(\theta_h-\theta_\ell\right)\delta_0}{\overline{\theta}}\right]<\overline{\theta}^2\left[\frac{\left(\theta_h-\theta_\ell\right)\delta_0}{\overline{\theta}}-1\right] \tag{66}$$

$$\frac{\psi\left(1-\overline{\theta}\right)}{\omega\left(1-\pi\right)}<\frac{\overline{\theta}^2\delta_0}{2\left(1-\delta_0\right)} \tag{67}$$

A sufficient condition for (64) is $\delta_0 \leq \frac{1}{2}$ which is still consistent with $\delta_0 \in \left(\frac{1}{3}, \frac{2}{3}\right)$. Condition (65) is just another positive upperbound on θ_ℓ provided $\pi > \frac{1}{1+2\delta_0}$. In other words, (65) is satisfied by θ_ℓ sufficiently low and π sufficiently high. Condition (66) will be true for $\frac{\psi}{\omega}$ sufficiently low if $\left(\theta_h-\theta_\ell\right)\delta_0 > \overline{\theta}$ or, equivalently, $\theta_\ell < \left[1-\frac{1}{\delta_0+\pi}\right]\theta_h$ with $\pi > 1-\delta_0$ which again means θ_ℓ sufficiently low and π sufficiently high. Finally, condition (67) is clearly satisfied by $\frac{\psi}{\omega}$ sufficiently low. $\square$ $\blacksquare$

Proof of Proposition 6

Evaluate (27) at $\alpha=\overline{\theta}$ then subtract (48) to get:

$$\xi_j-\xi_k \overset{sign}{=} \left[\frac{\left(1+i_A\right)^2-1}{1-\pi}-\frac{\overline{\theta}\left(1-\delta_0\right)}{\psi}\right]+2\left[\frac{\left(1+i_A\right)^2-1}{1-\pi}-i_L^h\right]$$

The expression in the first set of square brackets is positive by condition (33). The expression in the second set of square brackets is proportional to μ_j. The proof of Proposition 5 established $\mu_j > 0$. Therefore, $\xi_j > \xi_k$ at $\alpha=\overline{\theta}$.

Now consider total credit:

$$TC \equiv 1-R_j-R_k$$

Use market clearing as per (12) to replace R_j+R_k:

$$TC = 1-\overline{\theta}X_j-\theta_h X_k+\psi i_L^h$$

Use (24) and (25) to replace X_j and X_k:

$$TC = 1-\overline{\theta}-\left(\theta_h-\overline{\theta}\right)\delta_0+\delta_1\left(\theta_h-\overline{\theta}\right)\left(\xi_j-\xi_k\right)+\psi i_L^h$$

Proposition 5 showed $i_L^h\big|_{\alpha=\overline{\theta}} > i_L^h\big|_{\alpha=0}$. We also know $\xi_j=\xi_k=0$ at $\alpha=0$ and $\xi_j>\xi_k$ at $\alpha=\overline{\theta}$. Therefore, we can conclude $TC|_{\alpha=\overline{\theta}} > TC|_{\alpha=0}$.

Finally, we want to show that the loan-to-deposit ratios of big and small banks converge. The equilibrium has $\tau_j=1$, meaning that small banks move all DLPs (and the associated investments) off-balance-sheet. The loan-to-deposit ratio of the representative small bank is then $\lambda_j \equiv 1 - \frac{R_j}{X_j - W_j}$. The equilibrium also has $\tau_k = 0$, meaning that the big bank records everything on-balance-sheet. Its loan-to-deposit ratio is then $\lambda_k \equiv 1 - \frac{R_k}{X_k}$. Proposition 4 established $R_k > 0 = R_j$ at $\alpha = 0$ so it follows that $\lambda_k|_{\alpha=0} < 1 = \lambda_j|_{\alpha=0}$. To show convergence, we just need to show $\lambda_k|_{\alpha=\overline{\theta}} > \lambda_k|_{\alpha=0}$ since $\lambda_j|_{\alpha=\overline{\theta}} < \lambda_j|_{\alpha=0}$ follows immediately from equation (23). Use $X_j + X_k = 1$ along with the definition of λ_k to rewrite (12) as:

$$\psi i_L^h = \overline{\theta} + \left[\theta_h - \overline{\theta} - (1 - \lambda_k)\right] X_k - R_j$$

We know $i_L^h\big|_{\alpha=\overline{\theta}} > i_L^h\big|_{\alpha=0}$ so it must be the case that:

$$\left[\theta_h - \overline{\theta} - (1 - \lambda_k|_{\alpha=\overline{\theta}})\right] X_k|_{\alpha=\overline{\theta}} - R_j|_{\alpha=\overline{\theta}} > \left[\theta_h - \overline{\theta} - (1 - \lambda_k|_{\alpha=0})\right] X_k|_{\alpha=0}$$

Proposition 4 also established $\xi_j = \xi_k = 0$ at $\alpha = 0$. Substituting into equation (25) then implies $X_k = \delta_0$ at $\alpha = 0$ so:

$$\lambda_k|_{\alpha=\overline{\theta}} \frac{X_k|_{\alpha=\overline{\theta}}}{\delta_0} - \lambda_k|_{\alpha=0} > \underbrace{\frac{R_j|_{\alpha=\overline{\theta}}}{\delta_0} - \left[1 - \pi\left(\theta_h - \theta_\ell\right)\right]\left[1 - \frac{X_k|_{\alpha=\overline{\theta}}}{\delta_0}\right]}_{\text{call this } Z_6}$$

We have shown $\xi_j > \xi_k$ at $\alpha = \overline{\theta}$ so equation (25) also implies $\frac{X_k|_{\alpha=\overline{\theta}}}{\delta_0} \leq 1$ for any $\delta_1 \geq 0$. Therefore, $Z_6 \geq 0$ will be sufficient for $\lambda_k|_{\alpha=\overline{\theta}} > \lambda_k|_{\alpha=0}$. If $\delta_1 = 0$, then $Z_6 \propto R_j|_{\alpha=\overline{\theta}} \geq 0$. If $\delta_1 = \omega$, then we can rewrite $Z_6 \geq 0$ as:

$$1 - \delta_0 - \omega\xi_k \geq \frac{1 - \pi\left(\theta_h - \theta_\ell\right)}{\overline{\theta}}\omega\left(\xi_j - \xi_k\right) \tag{68}$$

where ξ_j is given by (27) with $\alpha = \overline{\theta}$ and ξ_k is given by (48). Use these expressions to substitute out ξ_j and ξ_k then use equation (47) to substitute out i_L^h. Evaluate i_A at (49) and ϕ^* at (50) to rewrite (68) as:

$$\begin{aligned} &\frac{4\psi\left(1 - \overline{\theta}\right)\left(1 - \delta_0\right)}{\omega\overline{\theta}\left(1 - \pi\right)} + \underbrace{\overline{\theta}\left(2 - 3\delta_0\right) + \left(1 - \theta_h\right)\left(\frac{2}{\delta_0} - 3 - \delta_0\right)}_{\text{call this } \Delta(\delta_0)} \\ \geq\ & -\frac{\overline{\theta}^2}{4}\frac{\omega\left(1 - \pi\right)}{\psi\left(1 - \overline{\theta}\right)}\underbrace{\left[2\overline{\theta}\left(1 - 2\delta_0\right) + \left(1 - \theta_h\right)\left(\frac{4}{\delta_0} - 6 - 3\delta_0\right)\right]}_{\text{call this } \widetilde{\Delta}(\delta_0)} \end{aligned}$$

A sufficient condition for this is $\min\left\{\Delta\left(\delta_0\right), \widetilde{\Delta}\left(\delta_0\right)\right\} \geq 0$. Notice $\Delta'(\cdot) < 0$ and $\widetilde{\Delta}'(\cdot) < 0$. Also notice $\min\left\{\Delta\left(\frac{1}{2}\right), \widetilde{\Delta}\left(\frac{1}{2}\right)\right\} > 0$ and $\min\left\{\Delta\left(\frac{2}{3}\right), \widetilde{\Delta}\left(\frac{2}{3}\right)\right\} < 0$. Therefore, there is a threshold $\overline{\delta_0} \in \left(\frac{1}{2}, \frac{2}{3}\right)$ such that $\delta_0 \leq \overline{\delta_0}$ guarantees $Z_6 \geq 0$. ■

Appendix B – Deposit and DLP Demands

Here we sketch a simple household maximization problem which generates the demands in equations (1) and (2). There is a continuum of ex ante identical households indexed by $i \in [0, 1]$. Each household is endowed with X units of funding. Let D_{ij} and W_{ij} denote the deposits and DLPs purchased by household i from bank j, where:

$$\sum_j \left(D_{ij} + W_{ij}\right) \leq X \tag{69}$$

Assume that buying W_{ij} entails a transaction cost of $\frac{1}{2\omega_0}W_{ij}^2$, where $\omega_0 > 0$. As per the main text, the interest rate on the DLP is zero if withdrawn early and ξ_j otherwise. The interest rate on deposits is always zero and the average probability of early withdrawal is $\overline{\theta}$. The household requires subsistence consumption of X in each state, above which it is risk neutral. If the household were to bypass the banking system and invest in long-term projects directly, it would fall below subsistence in the state where it needs to liquidate early since long-term projects cannot be liquidated early. Therefore, the household does not invest directly. Instead, it chooses D_{ij} and W_{ij} for each j to maximize:

$$\sum_j \left(D_{ij} + \left[1 + \left(1 - \overline{\theta}\right)\xi_j\right] W_{ij} - \frac{W_{ij}^2}{2\omega_0}\right)$$

subject to (69) holding with equality.[26] The first order condition with respect to W_{ij} is:

$$W_{ij} = \left(1 - \overline{\theta}\right)\omega_0 \xi_j \tag{70}$$

Substituting (70) into (69) when the latter holds with equality gives the household's total deposit demand, $D_i \equiv \sum_j D_{ij}$. The household is indifferent about the allocation of D_i across

[26] Here is how to recover the two-point distribution of idiosyncratic bank shocks in Section 2 from the household withdrawals. Each household has probability θ_ℓ of being hit by an idiosyncratic consumption shock at $t = 1$ and having to withdraw all of its funding early. This results in each bank losing fraction θ_ℓ of its deposits and DLPs at $t = 1$. Then $\theta_h - \theta_\ell$ of the remaining $1 - \theta_\ell$ households observe a sunspot and withdraw all of their funding from $1 - \pi$ banks at $t = 1$. The $\theta_h - \theta_\ell$ households and $1 - \pi$ banks involved in the sunspot are chosen at random. Note $\overline{\theta} \equiv \pi\theta_\ell + (1 - \pi)\theta_h$.

banks so we assume that it simply allocates D_i uniformly. For J banks, this yields:

$$D_{ij} = \frac{X}{J} - \frac{\left(1-\overline{\theta}\right)\omega_0}{J}\xi_j - \frac{\left(J-1\right)\left(1-\overline{\theta}\right)\omega_0}{J}\frac{1}{J-1}\sum_{x\neq j}\xi_x \tag{71}$$

With a unit mass of ex ante identical households, $W_j = W_{ij}$ and $D_j = D_{ij}$. As J approaches a unit mass of equally-weighted banks, (70) and (71) belong to the family of functions specified by (1) and (2).

Appendix C – Benchmark with Aggregate Shock

Consider the benchmark model (only price-taking banks) in Section 2 but with an aggregate interbank shock. In particular, the interbank rate is i_L^ℓ with probability π and i_L^h with probability $1-\pi$. The expected interbank rate is $i_L^e \equiv \pi i_L^\ell + (1-\pi)\, i_L^h$. We will specify how i_L^ℓ and i_L^h are determined shortly. In the meantime, banks take both as given.

The objective function of the representative bank simplifies to:

$$\Upsilon_j = (1+i_A)^2\,(X_j - R_j) + (1+i_L^e)\,R_j - \left[X_j + i_L^e\overline{\theta}X_j + \left(1-\overline{\theta}\right)\xi_j W_j\right] - \tfrac{\phi}{2}X_j^2$$

This is identical to the benchmark model except with the expected interbank rate i_L^e instead of the deterministic i_L. Therefore, the first order conditions are still given by equations (7) to (9) but with i_L^e in place of i_L.

The goal is to show that i_L^e is always highest at $\alpha = 0$. The proof follows Proposition 3 but, to proceed, we must replace the deterministic market clearing condition (equation (4)) with conditions for each realization of the aggregate shock. We model the shock as a shock to the aggregate demand for liquidity at $t = 1$. In particular, aggregate liquidity demand is $\overline{\theta}X - \varepsilon$ with probability π and $\overline{\theta}X$ with probability $1-\pi$, where $\varepsilon > 0$. The interbank rates are then i_L^ℓ and i_L^h respectively. To avoid liquidity shortages, we need these rates to satisfy:

$$R_j + \psi i_L^\ell \geq \overline{\theta}X - \varepsilon \tag{72}$$

$$R_j + \psi i_L^h \geq \overline{\theta}X \tag{73}$$

The equilibrium i_L^h solves (73) with equality. If $i_L^h \leq \frac{\varepsilon}{\psi}$, then we can set $i_L^\ell = 0$. Otherwise, the equilibrium i_L^ℓ solves (72) with equality.

Let i^e_{L0} denote the expected interbank rate at $\alpha = 0$ and let $i^e_{L1}(\rho)$ denote the expected interbank rate at some $\alpha > 0$. Using (72) and (73), we can write:

$$i^e_{L1}(\rho) = \frac{\overline{\theta} X}{\psi} - \frac{R_{j1}(\rho)}{\psi} - \frac{\pi}{\psi} \min\left\{\overline{\theta} X - R_{j1}(\rho), \varepsilon\right\} \tag{74}$$

where $R_{j1}(\rho)$ is reserve holdings at the $\alpha > 0$ being considered. The proof of $i^e_{L1}(\rho) \leq i^e_{L0}$ proceeds by contradiction. In particular, suppose $i^e_{L1}(\rho) > i^e_{L0}$. Then (7) implies $\mu_j > 0$ at $\alpha = 0$. Complementary slackness then implies $R_j = 0$ at $\alpha = 0$ so we can write:

$$i^e_L = \frac{\overline{\theta} X}{\psi} - \frac{\pi}{\psi} \min\left\{\overline{\theta} X, \varepsilon\right\} \tag{75}$$

Subtract (75) from (74) to get:

$$i^e_{L1}(\rho) = i^e_{L0} - \frac{R_{j1}(\rho)}{\psi} + \frac{\pi}{\psi}\left[\min\left\{\overline{\theta} X, \varepsilon\right\} - \min\left\{\overline{\theta} X - R_{j1}(\rho), \varepsilon\right\}\right]$$

There are three cases. If $\varepsilon \leq \overline{\theta} X - R_{j1}(\rho)$, then:

$$i^e_{L1}(\rho) = i^e_{L0} - \frac{R_{j1}(\rho)}{\psi}$$

If $\overline{\theta} X - R_{j1}(\rho) < \varepsilon < \overline{\theta} X$, then:

$$i^e_{L1}(\rho) = i^e_{L0} - \frac{1-\pi}{\psi} R_{j1}(\rho) - \frac{\pi}{\psi}\left(\overline{\theta} X - \varepsilon\right)$$

If $\overline{\theta} X \leq \varepsilon$, then:

$$i^e_{L1}(\rho) = i^e_{L0} - \frac{1-\pi}{\psi} R_{j1}(\rho)$$

In each case, $i^e_{L1}(\rho) > i^e_{L0}$ would require $R_{j1}(\rho) < 0$ which is impossible. ■

Appendix D – Supplementary Material on China

Reforms We start with the market-oriented reforms initiated in the 1990s. The first effect of these reforms was to make the Big Four much more profit-driven. All went through a major restructuring in the mid-2000s and are now publicly listed. The government and the Big Four still have ties which limit how intensely the Big Four compete against each other (e.g., the government holds controlling interest and appoints bank executives) but the government is no longer involved in decisions at the operational level. The effect of the reforms has been striking. The average non-performing loan ratio of the Big Four, which had ballooned to 30% by the early 2000s, has remained around 2% in recent years. Combined profits also grew 19% annually from 2007 to 2014 to reach an unprecedented USD 184 billion in 2014. Individually, the banks in China's Big Four now constitute the first, second, fourth, and seventh largest banks in the world as measured by total assets.[27]

The second effect of the reforms was entry of small and medium-sized commercial banks.[28] These banks are still individually small when compared to the Big Four. For example, average deposits for the JSCBs were only 17% of average deposits for the Big Four in 2013. As a group though, small and medium-sized banks have chipped away at the Big Four's deposit share. In 1995, the Big Four held 80% of deposits in China. By 2005, they held 60%. The Big Four now account for roughly 50% of traditional deposits, meaning that their deposit share declined even after the major restructuring of the mid-2000s. To put this into perspective, big state-owned firms in the industrial sector did not experience a similar post-restructuring decline in market share. As we will see in the calibration, our model can account for the post-restructuring decline of the Big Four.

Regulations Turning now to China's regulatory environment, let us elaborate on the evolution of reserve requirements, deposit rate regulation, and capital requirements.

Reserve Requirements Official reserve requirements were 7.5% in 2005. There was a modest increase to 9.5% by early 2007 as part of a policy to sterilize the accumulation of foreign reserves without issuing central bank bills (Song, Storesletten, and Zilibotti (2014)). Reserve requirements were then rapidly increased in a manner complementary to the increasing frequency of CBRC's loan-to-deposit checks, hitting 15.5% by February 2010 before being raised another twelve times to reach 21.5% by December 2011.[29] China lifted the official loan-to-deposit component of its liquidity rules in late 2015 but reserve requirements remain high and loan-to-deposit restrictions can still technically be imposed via loan quotas so there has been little dismantling of off-balance-sheet infrastructures.

[27] http://www.relbanks.com/worlds-top-banks/assets

[28] These market-oriented reforms also paved the way for China's rising private sector. For more, see Allen, Qian, and Qian (2005), Hsieh and Klenow (2009), Song, Storesletten, and Zilibotti (2011), Brandt, Van Biesebroeck, and Zhang (2012), Lardy (2014), Hsieh and Song (2015), and the references therein.

[29] In practice, a reserve requirement is a narrower type of liquidity regulation than a loan-to-deposit cap because, unlike the latter, it specifies the form in which liquidity must be held. We abstracted from this difference in the model (i.e., liquid assets were reserves) but it is useful to point out the distinction here to avoid confusion about what the true liquidity requirement is. With a 75% loan-to-deposit cap, the liquidity requirement is always 25%: a reserve requirement of $x < 25\%$ just means that the bank has discretion over how to divide $25\% - x$ between reserves and other liquid (i.e., non-loan) assets.

Deposit Rate Regulation China has a long history of regulating interest rates on traditional deposits. Prior to 2004, the central bank simply set these rates. Downward flexibility was introduced in 2004 with no response: all banks stayed at the maximum allowable rate. Some upward flexibility was then introduced in 2012 and almost all banks for which we have systematic data responded by moving up to the new maximum. The highest deposit rate allowed by the central bank thus continued to serve as the effective deposit rate in China. In October 2015, China announced the removal of deposit rate ceilings. The response of deposit rates has been modest. This is consistent with the most interest-sensitive savings having already migrated to high-yielding WMPs.

Capital Requirements After CBRC was established in 2003, it introduced an 8% minimum capital adequacy ratio as per Basel I. The higher requirements of Basel III are currently being phased in. CBRC will require a minimum capital adequacy ratio of 11.5% for systemically important banks and 10.5% for all other banks by the end of 2018. The requirements were 9.5% and 8.5% respectively at the end of 2013. For comparison, the Big Four had an average capital ratio of 12.7% in 2013 while the average across all Chinese banks in Bankscope was 14.7%. Data from Bankscope also shows that, on average, China's small and medium-sized banks held more than the minimum capital requirement even before CBRC adopted the Basel framework in 2004. The Big Four have also exceeded minimum capital requirements since being restructured in the mid-2000s. Capital requirements were therefore not a binding constraint in China for the period we study.

Trust Companies As noted in Section 4.1, most of the funds raised through off-balance-sheet WMPs are invested with the help of lightly-regulated institutions called trust companies. The off-balance-sheet vehicles in our model encapsulate the accounting maneuvers used to get the money to these trusts. Figure 2 shows a near lockstep evolution of trust company assets under management and WMPs outstanding. Based on data from the China Trustee Association, the funding for roughly 70% of trust assets comes from money that has already been pooled together by other financial institutions. This is remarkably close to the proportion of WMPs that are not guaranteed: 70% in 2012 and 65% in 2013, according to data from CBRC.

Further evidence that trust companies are major recipients of WMP money comes from the fact that they have responded to attempts at WMP regulation. For example, in August 2010, CBRC announced that WMPs could invest at most 30% in trust loans. The composition of trust assets then changed from 63% loans at the end of 2010Q2 to 42% loans by the end of 2011Q3, with "long-term investments" replacing loans. On average, 70% of trust assets took the form of loans or long-term investments from 2010 to 2014. The long-term nature of trust company assets is also apparent from the fact that trusts issued products with an average maturity of 1.7 years when trying to pool money on their own during the first half of 2013.[30] In contrast, WMPs are short-term products, with a median maturity between 2 and 4 months since 2008. The funneling of WMP money to trust companies thus involves a maturity mismatch.

Banks that funnel WMP money through trusts typically instruct the trusts where to invest the funds. Data from the China Trustee Association indicates that the sectorial composition of trust company assets has become more even over time, with infrastructure and real estate projects losing ground to industrial and commercial enterprises.

Measures of Shadow Banking The Financial Stability Board defines shadow banking as "credit intermediation [that] takes place in an environment where prudential regulatory standards ... are applied to a materially lesser or different degree than is the case for regular banks engaged in similar activities" (FSB (2011)). The cooperation between banks and trusts discussed above satisfies this definition. First, it involves maturity transformation and thus constitutes banking in the sense of Diamond and Dybvig (1983). Second, it is funded by off-balance-sheet WMPs which are booked away from regulatory standards. In Section 5, we estimate that China's shadow banking system, as defined by off-balance-sheet WMPs, grew from a negligible fraction of GDP in 2007 to 16% of GDP in 2014.

To get a broader estimate of shadow banking, one can use the widely-cited data on total social financing constructed by China's National Bureau of Statistics (NBS). Social financing includes bank loans, corporate bonds, equity, and other financing not accounted for by traditional channels. Roughly one-third of other financing takes the form of undiscounted banker's acceptances.[31] Removing these acceptances then leaves the most shadowy part of other financing, namely loans by trust companies and firm-to-firm loans that use banks as trustees (entrusted loans). The NBS defines trust loans very narrowly so credit extended by trust companies in more obscure ways may actually be picked up in entrusted lending. To this point, Allen, Qian, Tu, and Yu (2015) find that firms which are required to disclose entrusted loans (in particular, publicly traded firms) accounted for 10% of total entrusted lending reported by China's central bank in 2013. In other words, the set of entrusted loans for which it is easiest to identify and exclude trust company involvement accounts for only a small fraction of entrusted lending. If trust and entrusted loans are grouped into one measure of shadow banking, then shadow banking grew from 5% of GDP in 2007 to 24% of

[30] Annual Report of the Trust Industry in China (2013).

[31] A banker's acceptance is basically a guarantee by a bank on behalf of a depositor. More precisely, the bank guarantees that the depositor will repay a third-party at a later date.

GDP in 2014. This 19 percentage point increase is very similar to the 16 percentage point increase estimated above using only off-balance-sheet WMPs.

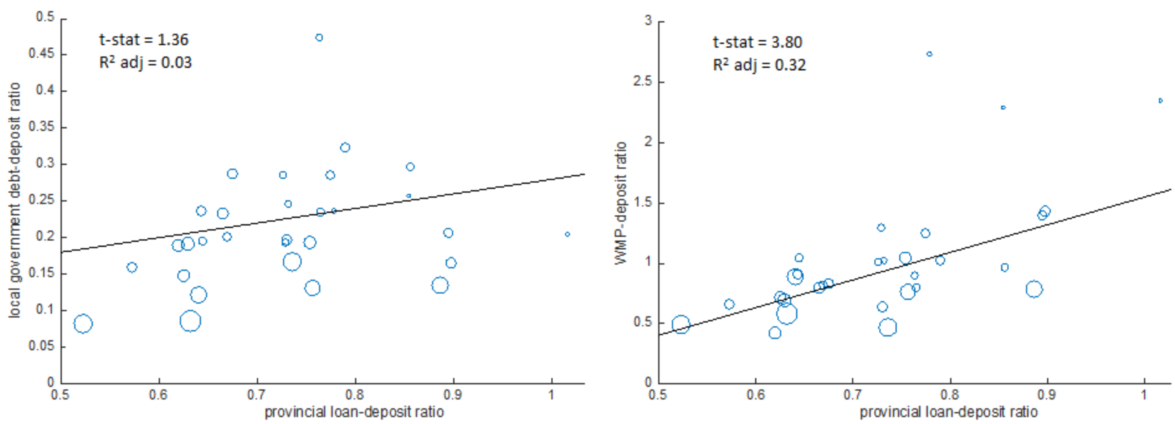
Fiscal Stimulus and Shadow Banking Recent work by Acharya, Qian, and Yang (2016) contends that fiscal stimulus increased the loan-to-deposit ratios of small banks. They argue that Bank of China (one of the Big Four) was particularly willing to provide stimulus loans and, as it raised deposits in order to make these loans, its smaller competitors were crowded out of the deposit market and forced to attract funding by issuing off-balance-sheet WMPs with high returns. We start by explaining why this story does not challenge our view that loan-to-deposit enforcement triggered the rise of off-balance-sheet WMPs by small banks. We will then discuss several results in their paper that corroborate results in ours.

First, had CBRC not begun stricter enforcement of the loan-to-deposit cap, small banks crowded out by the stimulus could have simply issued high-return *on*-balance-sheet WMPs to attract funding and continue lending. As long as the WMP is not classified as a traditional deposit, it is not subject to the central bank's deposit rate regulations. To this point, the return on 3-month on-balance-sheet WMPs averaged almost 90 basis points above the 3-month benchmark deposit rate from 2008 to 2014, with the spread widening to over 130 basis points in the second half of this sample.

Second, stimulus-related forces are not the main reason why small banks had high loan-to-deposit ratios around the time CBRC toughened its stance. The stimulus package was announced in late 2008 and funded from 2009 to 2010 so, if stimulus explains why small banks were constrained by the 75% loan-to-deposit cap, then small banks should have had loan-to-deposit ratios below 75% in 2007 and 2008. However, as shown by the average balance data in Figure 1, small banks had loan-to-deposit ratios above 75% during both of these years. Increasingly strict enforcement of the 75% cap by CBRC would have therefore bound on them even without any additional pressure from the stimulus package.

Third, when stimulus activity is measured using data from the borrowers, areas with the most stimulus activity were not necessarily those with the highest loan-to-deposit ratios. As documented in Bai, Hsieh, and Song (2016), most of the stimulus package had to be borrowed by local governments so it will be instructive to look specifically at local government debt. For each province in 2013, the left panel of Figure D.1 plots the stock of local government debt (as a fraction of deposits) against the provincial loan-to-deposit ratio. The latter is based on regional data from Wind which aggregates bank branches by province. Bigger provinces, as measured by deposits, are represented by bigger dots. For comparison, the right panel of Figure D.1 plots WMP batches (relative to deposits in trillions of RMB) against loan-to-deposit ratios. CBRC enforced the 75% loan-to-deposit cap at the bank level, not the branch level, so a distribution of provincial loan-to-deposit ratios exists. However, individual branches would still have needed an infrastructure that could accommodate sudden requests from their parent banks to decrease on-balance-sheet loan-to-deposit ratios as the parents worked around CBRC's enforcement. The positive correlation in the right panel of Figure D.1 is statistically significant whereas the positive correlation in the left panel is not. In other words, provinces with higher loan-to-deposit ratios in 2013 tended to have more WMP issuance but did not necessarily have more local government debt.[32]

Figure D.1



Source: Bai, Hsieh, and Song (2016) and Wind. Data is for 2013.

Before proceeding, we note that several results in Acharya, Qian, and Yang (2016) are also complementary to our story. First, they find that the loan-to-deposit ratio is more important than the capital adequacy ratio in explaining WMP patterns. This is clearly consistent with our view. Second, they find a positive correlation between the interbank interest rate and WMP issuance by small banks with high loan-to-deposit ratios. Our model generates this correlation. We show that the Big Four tighten the interbank market as part of their optimal response to competition from small bank WMPs. Therefore, as small banks constrained by loan-to-deposit regulation issue WMPs, big banks increase interest rates on the interbank market, generating the positive correlation observed in the data. Third, they find that big banks with more WMPs coming due ask for higher interest rates on the interbank market. In our model, a higher interbank rate lowers the returns that small banks offer to attract off-balance-sheet funding, which would make it easier for the big bank to roll over a large batch of WMPs without having to offer very high returns.

[32] A similar message emerges if the loan-to-deposit metric is defined as the change in outstanding loans between 2008 and 2013 divided by deposits in 2013.

Monetary Policy and Shadow Banking Recent work by Chen, Ren, and Zha (2016) contends that shadow banking in China involves small banks using securities called accounts receivable investments (ARIs) to bring entrusted loans on balance sheet. The authors attribute the rise of ARIs to monetary tightening which they define as lower M2 growth. In the quantitative analysis of Section 5.3, we find that shocks to liquidity regulation, distinct from shocks to the supply of liquidity by the central bank, play the dominant role in explaining the comovement of key interest rates in China, namely the interbank interest rate and the returns on WMPs.

It is also important to note that small banks held negligible amounts of ARIs prior to 2012. The rise of ARIs was thus predated by the rise of off-balance-sheet WMPs which, as explained earlier in this appendix, constitute shadow banking. Our paper thus tackles the foundations of China's shadow banking system.

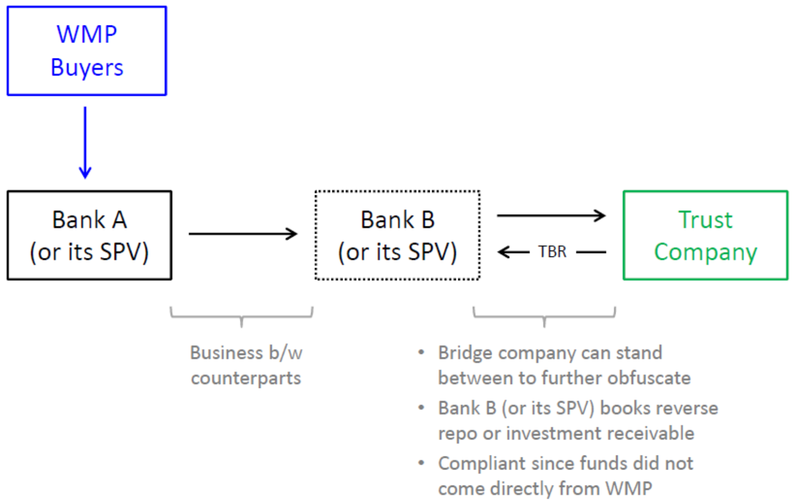
In fact, once these foundations are understood, the use of ARIs after 2012 follows quite naturally from the ideas in our paper.[33] In March 2013, CBRC announced that WMPs could invest at most 35% in non-standard debt assets (e.g., trust assets). Banks using off-balance-sheet WMPs to circumvent liquidity regulation thus needed to find a less direct way to funnel WMP money into trust companies. The result was the counterpart business illustrated in Figure D.2. In short, money is channeled from WMPs to trusts in two individually compliant steps. The WMP issuer first places WMP money in another bank (or bank-affiliated off-balance-sheet vehicle) which we will call Bank B. The WMP's return is tied to interest earned on this placement so the WMP is said to be backed by interest rate products, not trust assets. However, trust companies appear in the next step. In particular, they issue trust beneficiary rights (TBRs) to Bank B who then uses the cash flows from those rights to pay the placement interest. Balance sheet data indicate that banks report TBR holdings either as an investment receivable (ARI) or as collateral in an "offline" reverse repo.[34] In other words, ARIs are part of the accounting maneuvers used by banks and trusts in response to CBRC's crackdown on direct bank-trust cooperation. At least part of the rise of ARIs after 2012 was therefore a continuation of the regulatory arbitrage that led to the rise of off-balance sheet WMPs before 2012.

[33] The explanations that follow extend a discussion in our early draft, "The Rise of China's Shadow Banking System," 2015.

[34] Offline transactions are ones which do not go through the China Foreign Exchange Trade System.

Figure D.2

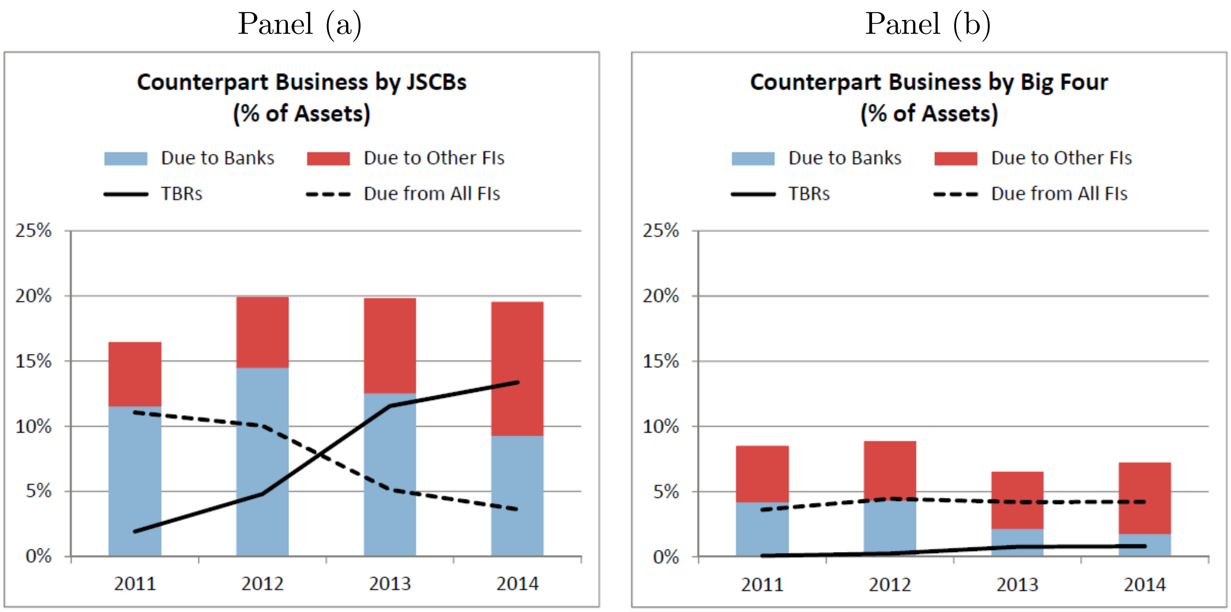
Business with Counterparts



Notes: TBR stands for trust beneficiary right; SPV is an off-balance-sheet vehicle

Figure D.3 reveals that the counterpart business just described is being facilitated by small banks. Specifically, the role of Bank B in Figure D.2 is played by joint-stock banks, not by banks in the Big Four. This helps explain the prevalence of ARIs on small bank balance sheets. Figure D.3 shows a dramatic rise in TBR holdings among joint-stock banks when CBRC cracked down on direct bank-trust cooperation in 2013. There was no similar rise in TBR holdings among the Big Four. Figure D.3 also shows that the joint-stock banks accommodated their increase in TBR holdings by keeping fewer balances at banks and other financial institutions (dashed black line). In other words, the JSCBs did not sacrifice loans in order to hold TBRs. The JSCBs' substitution away from balances at banks is also visible in the blue bars in Figure D.3: both the JSCBs and the Big Four have experienced decreases in balances owed to banks. However, unlike the Big Four, the JSCBs have attracted sufficiently more balances from non-bank financial institutions (red bars). Off-balance-sheet vehicles that hold unguaranteed WMPs would be classified as non-bank financial institutions. Effectively, the off-balance-sheet vehicle of one JSCB places money with another JSCB. The other JSCB then uses returns from its TBR holdings to pay interest on the placement. This is exactly the counterpart business illustrated in Figure D.2. In principle, even more counterpart business could be occurring between the vehicles themselves (e.g., vehicles can hold TBRs and placements can occur between vehicles).

Figure D.3



Source: Bank annual reports. The graphs report domestic balances only.

Appendix E – Money Multiplier Calculation

A standard money multiplier calculation will help assess the contribution of China's RMB 4 trillion fiscal stimulus package to (i) more aggressive lending by the Big Four and (ii) China's aggregate credit boom.

The size of the stimulus is S and the fraction to be financed by the Big Four is q. To finance qS, the Big Four make a one-time transfer of qS from excess reserves to loans. We will treat the Big Four as a closed system, meaning that their loans do not end up in deposit accounts at the small banks. With a currency ratio of c and a reserve ratio of r, the multiplier process increases loans and deposits at the Big Four by $\frac{qS}{1-(1-r)(1-c)}$ and $\frac{(1-c)qS}{1-(1-r)(1-c)}$ respectively. We use a conservative currency ratio ($c = 0.05$) so as not to understate the effect of the stimulus package on Big Four loans. Since we want an indication of the effects of the stimulus package had nothing else changed, we set $r = 0.35$, recalling from Figure 1 that the loan-to-deposit ratio of big banks averaged just over 0.6 between 2005 and 2008.

For the fraction of stimulus financed by the Big Four, we use $q = 0.65$. The Big Four had a market share (as measured by deposits) of roughly 55% in 2007. They may have been willing to pitch in a bit more, hence $q > 0.55$, but there is little to suggest they were expected to finance a disproportionate share of the stimulus package. The details of the package were such that the central government could only fund up to 33% of the planned investment/expenditure. The rest of the funds were to be borrowed by local governments. In modern-day China, the central government exercises only indirect control over the Big Four (e.g., through top personnel decisions) and can exercise exactly the same control over the JSCBs since they also operate nationally. Local governments have no administrative control over commercial banks. Their only leverage is economic (e.g., where to put local government savings) and it tends to work better with smaller/locally-operating banks such as city banks. In short, local governments are simply not powerful enough to force the Big Four (or even the JSCBs) to lend huge amounts of money that they otherwise would not have lent. For this reason, the fraction of the stimulus that we attribute to the Big Four is guided by their market share.

The results of our calculation suggest that China's stimulus package can account for up to RMB 6.8 trillion of new loans and up to RMB 6.5 trillion of new deposits at the Big Four since the end of 2008. Taking these amounts out, loans and deposits at the Big Four would have grown at annualized rates of 12.9% and 9.8% respectively from 2008 to 2014. The Big Four's loan-to-deposit ratio would have then increased from 0.57 in 2008 to 0.67 in 2014. This counterfactual estimate of what would have happened to the Big Four's loan-to-deposit ratio absent stimulus is similar to what actually happened with the stimulus (an increase from 0.57 in 2008 to 0.70 in 2014 as per Figure 1).

We now extend the calculation to estimate how much of the overall increase in China's credit-to-savings ratio can be reasonably explained by stimulus alone. Suppose the remaining $(1-q)\,S$ of stimulus was financed by small banks. This amount will also go through the multiplier process, except with a lower reserve ratio (call it $\widetilde{r}$) since small banks have higher loan-to-deposit ratios than the Big Four. We set $\widetilde{r} = 0.15$ based on the average balance data for 2007 in Figure 1. Combining the calculations for the small banks with the calculations for the Big Four, we find that the stimulus package explains around 40% of the 10 percentage point increase in China's credit-to-savings ratio since 2007.

Appendix F – Estimation Procedure

Let $m = 1, ..., 6$ index the empirical moments to be matched. The six moments are the six correlations in Table 2.

1. Bootstrap: Let N denote the total number of random samples generated by bootstrap. We set $N = 500$. Denote by $g_{m,n}$ the m^{th} moment in the n^{th} sample. We will target $\frac{1}{N}\sum_{n}^{N} g_{m,n}$, the m^{th} moment averaged across N samples.

2. Denote by Ω the vector of parameters to be estimated. Given Ω, we can simulate the model to generate the moments $g_m(\Omega)$. Denote by $\varepsilon_{m,n} = g_m(\Omega) - g_{m,n}$ the residual for moment m in sample n. Define the weighting matrix $(M \times M)$ as:

$$W = \frac{1}{N} \sum_{n}^{N} \varepsilon_{m,n} \varepsilon_{m,n}^{T}$$

3. Minimizing the weighted sum of the distance between the empirical and simulated moments:

$$\widehat{\Omega} = \arg\min_{\Omega} h(\Omega)' W^{-1} h(\Omega)$$

where $h(\Omega)$ is a vector with M elements and $h_m(\Omega) = g_m(\Omega) - \frac{1}{N}\sum_n^N g_{m,n}$.

4. We use two-step Simulated Method of Moments. We set W to the identity matrix in the first step and use W from the first-step as the weighting matrix for the second-step estimation.

5. Repeat the above exercise 100 times to calculate the standard errors of the estimated parameters.