Payments, Credit and Asset Prices*

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Abstract

This paper studies a monetary economy with two layers of transactions. In enduser transactions, households and institutional investors pay for goods and securities with payment instruments provided by banks. Endusers' payment instructions generate interbank transactions that banks handle with reserves or interbank credit. The model links the payments system and securities markets so that beliefs about asset payoffs matter for the price level, and monetary policy matters for real asset values.

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1 Introduction

This paper studies the joint determination of payments, credit and asset prices. The starting point is that, in modern economies, transactions occur in two layers. In the enduser layer, non-banks — for example, households, firms and institutional investors — trade goods and securities and pay for them using payment instruments supplied by banks. Payment instruments include not only short term demandable debt such as deposits, but also credit lines that can be drawn on demand such as credit cards. Credit lines also pay a key role in payment for securities. For example, institutional investors make use of sweep arrangements with their custodian banks. Participants in the triparty repo market have been obtaining intraday credit from the clearing banks.

A common denominator of different payment instruments is that banks commit to accept payment instructions from their clients. As a result of those payment instructions, transactions in the enduser layer generate interbank transactions in the bank layer. Perhaps the most obvious example is direct payment out of a bank deposit account by cheque or wire transfer: payments between customers of different banks generate interbank transfers of funds. In many securities markets, transactions are cleared by specialized financial market utilities such as clearinghouses that provide some netting of transactions. Institutional investors then settle netted positions with those utilities through payment instructions to their banks.

Interbank payments are often made with reserves, but may also be handled through various forms of short term credit. For example, in the United States, utilities like NSS and CHIPS allow for intraday netting of a share of interbank transactions, so only net positions are periodically settled with reserves. The central bank may also provide intraday overdraft credit to banks. Nevertheless, the bulk of interbank payments goes through gross settlement systems provided by central banks, such as Fedwire in the United States or Target in the Euro Area.

In the aftermath of recent financial crises, central banks have made unprecedented changes to the quantity as well as the price of reserves. On the one hand, several central banks dramatically increased the quantity of reserves relative to the value of transactions. These policy shifts have reduced the relative importance of interbank credit. For example, in the United States, the use of intraday overdraft as well as interbank overnight Fed funds borrowing have essentially disappeared. On the other hand, a number of central banks have begun charging negative nominal interest rates for the use of reserves.

This paper proposes a stylized model of an exchange economy with two layers of transactions. The endusers are households and institutional investors who must pay for some goods and securities via prearranged payment instruments supplied by a competitive banking sector. There is no currency, but banks use reserves supplied by the government as well as interbank credit to handle payment instructions generated by enduser transactions. The model determines asset prices, the nominal price level and agents’ portfolios as a function of government policy as well as investor beliefs about asset payoffs. We use it to think about the interaction of securities markets and the payment system as well as the effect of recent policy shifts.

The model is based on three premises. First, both banks and the government incur costs of leverage that decline with a measure of collateral. When banks issue deposits or provide loan commitments, they must expend effort to convince their enduser clients that those payment
instruments can be relied on when needed. The larger and safer is the bank’s asset portfolio, the fewer resources are required to provide credible payment instruments. Similarly, when the government issues reserves or other debt, resources are needed to convince lenders of the commitment to repay, and less so if debt is small relative to the tax base which serves as collateral for the government.

We further assume that the supply of payment instruments – the only media of exchange for endusers – depends on banks’ leverage choice, and that institutional investors require payment instruments to trade. Both premises are reflected in the quantity equation

\[ PT = \bar{v}(D + L), \]

where the total volume of transactions \( T \) includes institutional investors’ securities purchases, and payment instruments consist of deposits \( D \) and loan commitments \( L \) provided by banks.

The quantity equation illustrates two linkages between the payments system and securities markets, and hence asset values, and the nominal price level. Consider first the supply of payment instruments: if an event in securities markets – such as an increase in uncertainty about asset payoffs – lowers asset values, it also lowers the value of banks’s collateral and makes the production of payment instruments more costly. A decline in \( D + L \) then puts downward pressure on the price level.

Second, the event also lowers transactions by institutional investors and hence the demand for payment instruments. As fewer payment instruments are used in securities markets, more can be used to pay for goods, thus pushing the price level up. The main conclusion is that, in an economy with two layers, shocks to securities markets are transmitted to the payments system. Moreover, the details of financial structure, including the use of payment instruments by institutional investors and the scope of netting arrangements – are important in order to assess the effects of asset market shocks on inflation. Here

The challenge for policy in our economy is how to handle the volume of transactions \( T \) with minimal leverage cost. The latter comprises resources used by banks to produce payment instruments \( D + L \) for the enduser layer, as well as resources used by the government to issue reserves for the bank layer. The government has two tools. First, it controls the real return on reserves: it can set the nominal rate on reserves, and the inflation rate follows from the growth rate of its nominal liabilities, which serve as collateral for banks. Second, the government can trade in securities markets to change the mix of collateral available to banks.

The government can use its tools to select one of two policy regimes. Reserves are scarce if banks do not always have sufficient reserves to handle all interbank payments but instead may have to turn to the overnight credit market to fund themselves. The liquidity benefit of reserves then generates a spread between the overnight interest rate and the interest rate on reserves. Reserves become scarce is the real return on reserves is sufficiently low, that is, there is a large enough tax on reserves. As long as reserves are scarce, open market purchases that exchange short term debt for reserves change the collateral mix.

In contrast, reserves are abundant if the real quantity of reserves is sufficiently large relative to the volume of transactions that overnight borrowing is never needed. Once reserves lose their liquidity benefit, overnight loans and reserves become perfect substitutes and earn the same interest rate. Reserves become abundant when the real return on reserves is sufficiently high.
With abundant reserves, only unconventional policy that exchanges reserves for lower quality collateral can change the collateral mix.

Which regime is better depends on the relative leverage costs of banks versus the government. If the government can borrow more cheaply than banks, then it makes sense to move to abundant reserves, as several central banks have done recently. An extreme approach would be a move to narrow banking. In contrast, if the government has trouble to credibly commit to a path for nominal debt, then it is beneficial to have banks rely more on collateral other than government debt (or reserves) in order to produce payment instruments. Since the optimal system depends on the quality of collateral, it may make sense to switch between regimes over time in response to asset market events.

The availability of two separate policy tools implies that the stance of policy cannot be easily summarized by a single variable, such as the nominal interest rate. For example, when reserves are scarce, the government can change either the collateral mix or the real return on reserves to move the nominal interest rate. However, the effect on real interest rates and inflation is generally different. The reason is that asset values reflect not only liquidity benefits — as in many monetary models — but also collateral benefits. Policy affects interest rates by altering both benefits.

The model assumes that households have linear utility, and that banks and other financial firms maximize shareholder value and operate under constant returns to scale. Markets are competitive and all prices are perfectly flexible. There are however two sources of friction in financial markets. First, payment instruments and reserves are more liquid than other assets; formally, they relax liquidity constraints in the enduser and bank layer, respectively. Second, leverage is costly for both firms and the government. Both frictions matter for portfolio choice, the production of payment instruments and pricing. However, banks and other financial firms are free to adjust equity. The role of frictions is therefore not to amplify shocks that temporarily lower financial firms’ capital. Instead, we study steady states in which changes in expectations and the monetary policy regime have permanent effects.¹

The model can be interpreted as describing the subset of worldwide transactions in a currency, rather than the closed economy of a country. The former interpretation is appropriate for economies like the United States that have banking systems and financial markets tightly integrated with those of other countries. We thus think of households in our models as agents who pay for goods out of dollar deposit accounts, while institutional investors may include foreign firms who obtain credit or payments from banks in terms of dollars. With this perspective in mind, the model can be used to think about how events in worldwide asset markets may affect nominal prices in the US.

A useful benchmark for comparison is the monetary asset pricing model of Lucas (1980). In that model, a given amount of goods market transactions occurs every period and is paid for with currency. Money is not needed in asset markets, so $T$ does not include asset market transactions. With positive nominal rates, the nominal price level follows from a standard quantity equation: if more currency is available to pay for the same amount of goods, then the price level rises. Returns on nominal and real assets are linked by the Fisher equation and differ

¹This approach also makes the model tractable – despite the presence of heterogeneous agents it lends itself to simple graphical analysis.
by compensation for inflation. In steady state, real asset returns are equal to the representative agent’s discount rate. The real values of money and other assets are thus determined separately.

For the baseline version of our model, we follow Lucas in positing a representative household, a fixed amount of goods market transactions, and no need for money in asset markets. However, transactions $T$ are paid for with payment instruments – deposits and credit lines – provided by banks. Since banks face leverage costs, the equilibrium quantity of payment instruments $D + L$ depends on the value of assets that banks can purchase to back these instruments. This creates a link between real asset values and the nominal price level. For example, if there is an increase in expected asset payoffs, banks compete to supply more payment instruments. As more instruments become available to pay for the same amount of transactions, the price level rises.

Nominal and real returns in our model are also linked by the Fisher equation. Steady state real returns, however, are not equal to the representative agent’s discount rate. This is because banks receive collateral benefits from assets they invest in to back the payment instruments they produce and thus bid up the prices of those assets. In particular, the real interest rate on short term credit is so low in equilibrium that the representative household chooses not to lend short term. This is true whether or not reserves are scarce.

More generally, abundance of reserves does not mean that households and other endusers treat all assets as perfect substitutes. Indeed, even when reserves and short term debt are perfect substitutes, both convey a collateral benefit that is valuable to banks, but not to households. At the same time, payment instruments retain their liquidity benefit to households. Since payment instruments require bank leverage, abundance of reserves does not drive this benefit to zero.

The fact that prices of assets held by banks (as well as institutional investors who borrow from banks) incorporate both collateral and liquidity benefits implies that monetary policy can permanently affect real asset prices and portfolios. Outside the liquidity trap, the central bank can change the asset mix towards more or less liquid bank assets. As long as reserves are more liquid than short credit, conventional open market purchases lower the need for interbank credit and the real interest rate. In the liquidity trap, the asset mix does not matter: asset purchases have real effects only if they change the overall value of bank assets, for example because the central bank buys at prices that are higher than what others would pay.

In the baseline model, the quantity of real payment instruments and securities held by banks is held fixed. In two extensions, we introduce institutional investors whose demand for loans or payment instruments responds to changes in interest rates. We first consider carry traders who hold real assets and borrow against those assets using short term credit supplied by banks. Carry traders have no demand for payment instruments, but supply collateral in the form of short term loans to banks. An example is asset management firms who finance securities holdings via repurchase agreements.

The key new feature in an economy with carry traders is that the price level now depends on carry traders’ demand for loans. For example, a decrease in uncertainty increases the demand for loans and hence the quantity of collateral for banks, the supply of payment instruments and the price level. An asset price boom can thus be accompanied by inflation even if the supply of reserves as well as the amount of goods transacted remains constant and banks hold no uncertain securities themselves. Moreover, monetary policy that lowers the real short term
interest rate lowers carry traders’ borrowing costs and boosts the aggregate market by allowing more leverage.

The second extension introduces *active traders* who hold not only securities but also payment instruments, since they must occasionally rebalance their portfolio using cash payments. One example is asset management firms who sometimes want to exploit opportunities quickly before they can sell their current portfolio. Active traders’ portfolio choice responds to the deposit interest rate offered by banks and the fee for credit lines they charge: if payment instruments are cheaper, active traders hold more of them, and the value of their transactions is higher. The strength of their response depends importantly on how much netting takes place among active traders through intraday credit systems.

The key new feature in an economy with active traders is that the price level now depends on active traders’ demand for payment instruments. For example, a decrease in uncertainty increases their demand for deposits and credit lines. As more of payment instruments provided by banks are used in asset market transactions, fewer instruments are used in goods market transactions and the price level declines. During an asset price boom, we may thus see low inflation even if the supply of reserves increases. Moreover, monetary policy that lowers the real short term interest rate lowers active traders’ trading costs and further boosts the aggregate market.

We model an increase in uncertainty perceived about future cash flows as a shift towards more pessimistic beliefs, motivated by models of ambiguity averse preferences. The model describes links between the payments system and asset markets that make shocks travel both directions. An increase in uncertainty that lowers securities prices through an increase in uncertainty premia makes it more costly for banks to create payment instruments and generates deflation. At the same time, monetary policy can lower real interest rates and further boost asset prices by making it cheaper for investors to trade or build leveraged positions.

*Related literature* to be written

The paper is structured as follows. Section 2 describes the model. Section 3 looks at the baseline model that features only households and banks. It shows how steady state equilibria can be studied graphically and considers different monetary policy tools. Section 4 introduces uncertainty and studies the link between the payment system and securities markets. It also extends the model to accommodate institutional investors as a second group of endusers.

## 2 Model

Time is discrete, there is one good and there are no aggregate shocks. A representative household consumes an endowment of goods as well as fruit from trees. The total amount of goods available for consumption is constant. The representative household also owns competitive financial firms. In this section, the only financial firms are banks who issue payment instruments; below we also introduce different types of asset management companies. All financial firms issue equity and participate in tree and credit markets along with the household itself.
Layers and frictions

The model describes transactions and asset positions in both the enduser layer and the bank layer. In the enduser layer, households and nonbank financial firms trade goods and assets. In the bank layer, banks trade securities and also borrow from and lend to each other. A key connection between layers is that enduser transactions generate payment instructions to banks.

The model incorporates two frictions. First, to capture the costs of supplying payment instruments and debt as well as the benefits of safe collateral, we assume that there is a cost of leverage for banks as well as other financial firms. The government similarly faces a cost of leverage, with the tax base serving as collateral. Second, to capture why some assets provide liquidity benefits or costs, we impose liquidity constraints in both the enduser and bank layer.

In the enduser layer, liquidity constraints require payment instruments supplied by banks. We allow for both deposits held for one period and for credit lines arranged a period in advance. Those instruments are equivalent in equilibrium – both provide liquidity to endusers and both require costly leverage by banks. A binding enduser liquidity constraint thus generates a liquidity benefit of inside money.

Liquidity constraints in the bank layer require that interbank payments generated by customer withdrawals are paid with borrowed or unborrowed reserves. A binding bank liquidity constraint generate liquidity benefits for outside money (reserves) as well as interbank credit. It also generates a liquidity cost of providing payment instruments.

We assume that financial firms can be costlessly recapitalized every period and that their objective function exhibits constant returns to scale. As a result, a firm’s history does not constrain its future portfolio and capital structure decisions. As in Lagos and Wright (2005), the distribution of heterogeneous agents’ (here firms’) histories thus plays essentially no role in the model.

Assets

There are four different asset classes. Reserves serve as numeraire; they are issued by the government, pay a nominal interest rate \( i^R \) and are held only by banks. Overnight credit pays an interest rate \( i \); it is less liquid than reserves as funds lent out at date \( t \) cannot be used to handle customer withdrawals at date \( t+1 \).

Trees are infinitely lived assets that each pay an exogenous quantity of goods \( x_t \) in period \( t \). Trees differ only in who is allowed to invest in them, as detailed below. For now, we index trees by types \( j \in [0,1] \). The nominal price of a type \( j \) tree is denoted \( Q^j_t \), and the nominal value of its fruit is \( P_t x_t \), where \( P_t \) is the nominal price level.

2.1 Households

Households have linear utility, discount the future at rate \( \delta = -\log \beta \) and receive an endowment \( \Omega_t \) every period. Households enter the period with deposits \( D^h_t \) and outstanding credit lines \( L^h_t \); they buy consumption \( C_t \) at the nominal price \( P_t \) measured in units of reserves. Their liquidity constraint is

\[
P_t C_t \leq \bar{v}(D^h_t + L^h_t),
\]
where $\pi$ is a fixed parameter that determines the velocity of money available to households. A cash-in-advance approach helps zero in on the role of endogenous inside money. An alternative would be to use a money-in-the-utility approach.

The liquidity constraint allows for two types of nominal payment instruments. Deposits require investment one period in advance and earn the nominal interest rate $i^D_{t-1}$. Credit lines must be arranged one period in advance with a bank, but require no investment — they represent intraday credit extended by banks on demand. In exchange for the commitment to accept payment instructions, banks charge a fee $i^L_{t-1} L_t$ proportional to the (nominal) amount of credit.$^2$

The household budget constraint is

$$P_tC_t + i^L_{t-1} L_t^h = P_t Q_t + D_t^h (1 + i^D_{t-1}) - D_t^h + \int_0^1 ((Q_t^j + P_t x_t) \theta_{t-1}^j - Q_t^j \theta_t^j) dj$$

+ dividends + fees + government transfers.

Expenditure on goods as well as fees paid for credit lines must be financed through either (i) the sale of the endowment, (ii) changes in household asset positions in trees and deposits, or (iii) exogenous income from dividends, fees or government transfers, described in more detail below.

Households are allowed to invest in all trees $j \in [0, 1]$, but they cannot sell trees short, that is, we impose $\theta^j \geq 0$. We interpret the endowment as labor income (payoffs from human capital), while trees represent other long lived assets such as equity in nonfinancial firms or claims to housing services. Both trees and human capital are less illiquid in the sense that they cannot be used to pay for consumption. The difference between them is that human capital must be held by households, whereas trees can also be held by financial firms or the government, and their ownership affects the production of payment instruments. A key equilibrium outcome is where in the economy trees are held.

We think of our model period as a short period such as a day, and we do not study hyper-inflation periods, so that nominal and real rates of return are always small decimal numbers. We thus simplify formulas throughout by using the approximation $e^r = 1 + r$ for any small rate of return $r$, and setting any products of rates of return to zero. For example, with an inflation rate $\pi_t = \log \frac{P_t}{P_{t-1}}$, the real rate of return on deposits is $i^D_{t-1} - \pi_t$.

**Household choices**

Let $\omega_t$ denote the household’s marginal utility of wealth and let $\hat{\delta}_t = -\log(\beta \omega_{t+1}/\omega_t)$ denote minus the logarithm of the MRS of wealth next period for wealth this period.$^3$ The household first-order condition for type $j$ trees is

$$Q_t^j \geq e^{-\hat{\delta}_t} \left(Q_{t+1}^j + P_{t+1} x_{t+1}\right) \frac{P_t}{P_{t+1}}.$$

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$^2$We assume that an interest rate $i^D$ is earned on deposits regardless of whether they are spent, and that the fee $i^L$ is paid on credit lines regardless of whether they are drawn. These assumptions help simplify the algebra. More detailed modeling of the fee structure of different payment instruments is possibly interesting but not likely to be first order for the questions we address in this paper.

$^3$Here $\omega_t$ is the Lagrange multiplier on the date $t$ budget constraint (1). In a steady state, $\omega_t$ is constant over time and $\hat{\delta} = \delta$. More generally, the MRS of wealth may differ from $\delta$ if the liquidity cost of buying consumption goods changes over time.
The condition holds with equality if the household has a positive position in type $j$ trees. The rate of return on trees held by the household is $\delta_t$; such trees always exist in equilibrium since banks can hold only a subset of trees.

We focus on equilibria in which

$$i^D_t - \pi_{t+1} = \delta_t - i^L_t < \delta_t.$$  \hspace{1cm} (2)

The equality implies that households are indifferent between the two payment instruments. Indeed, households who invest in deposits must provide funds a period in advance on which they receive real return. Households who arrange a credit line can instead invest the funds in trees that yield $\delta$, but must then pay the fee $i^L$. The inequality says that payment instruments are costly. Households’ optimal choice is then to hold as few payment instruments as necessary, that is, the liquidity constraint binds.

2.2 Banks

The household owns many competitive banks. We describe the problem of a typical bank which maximizes shareholder value

$$\sum_{t=0}^{\infty} \exp \left( - \sum_{\tau=0}^{t-1} \delta_\tau \right) y^b_t.$$  \hspace{1cm} (3)

Here bank dividends $y^b_t$ are discounted at the rate $\delta_t$, as are payoffs from trees owned by households. Dividends can be positive or negative; the latter corresponds to recapitalization of the bank.

Liquidity management

The typical bank enters the period with deposits $D_t$, outstanding credit lines $L_t$ and reserves $M_t$. We want to capture the fact that enduser payment instructions may lead to payments between banks. For example, a payment made by debiting a deposit account or drawing a credit line may be credited to an account holder at a different bank. We thus assume that the typical bank receives an idiosyncratic withdrawal shock: an amount $\phi_t \bar{v}(D_t + L_t)$ must be sent to other banks, where $\phi$ is iid across banks with mean zero and cdf $G$. We also assume that $\phi$ is bounded above: the cdf $G$ is increasing only up to a bound $\bar{\phi}$ with $\bar{\phi} \bar{v} < 1$.

In the cross section, some banks draw shocks $\phi_t > 0$ and must make payments, while other banks draw shocks $\phi_t < 0$ and thus receive payments. Since $E[\phi] = 0$, any funds that leave one bank arrive at another bank; there is no aggregate flow into or out of the banking system. The scale of the required gross interbank payments depends on velocity in the enduser layer. In particular, if households almost never use their payment instruments, $\bar{\phi}$ is close to zero and few interbank payments are needed. The distribution of $\phi_t$ depends on the structure of the banking system as well as the pattern of payment flows among endusers.\footnote{The likelihood of payment shocks is the same across banks, regardless of bank size. Since the size distribution of banks is not determinate in equilibrium below, little is lost in thinking about equally sized banks. Alternatively, one may think about large banks consisting of small branches that cannot manage liquidity jointly but instead each must deal with their own shocks.}
Banks that need to make a transfer $\phi_t \bar{v}(D_t + L_t) > 0$ can send reserves they have brought into the period, or they can borrow from other banks. The bank liquidity constraint is

$$\phi_t \bar{v}(D_t + L_t) \leq (1 + \gamma) (M_t + F_{t+1}),$$  \hspace{1cm} (4)$$

where $F_{t+1} \geq 0$ is overnight borrowing and $\gamma \geq 0$ is a parameter that captures the efficacy of netting arrangements among banks.

If $\gamma = 0$, there is no netting: all interbank transfers must be made using either reserves $M_t$ or overnight credit $F_{t+1}$. More generally, $\gamma > 0$ allows banks to make more than one dollar of transfers per dollar of liquid funds $M_t + F_{t+1}$. We can think of those liquid funds as a downpayment in an intraday credit system. Below we describe a timing protocol to capture this idea and derive implications for the observed volume of intraday and overnight credit.

Suppose the marginal cost of overnight credit is larger than other sources of funding available to the bank. In this case – which we consider below – it is always better to take out as little overnight credit as necessary. Banks thus optimally choose a threshold rule: do not borrow overnight unless $\phi_t$ is so large that the withdrawal $\phi_t \bar{v}(D_t + L_t)$ exhausts both reserves and the intraday credit limit $\gamma M_t$ available by just paying down reserves.

For a bank that enters the period with reserves $M_t$, deposits $D_t$ and credit lines $L_t$, the liquidity constraint (4) implies a threshold shock

$$\phi^* = \frac{(1 + \gamma) M_t}{\bar{v}(D_t + L_t)}.$$  \hspace{1cm} (5)$$

We refer to $\phi^*$ as the liquidity ratio of a bank. It is inversely proportional to a money multiplier that relates the total value of payment instruments to the quantity of reserves.

If the liquidity constraint (4) binds, then reserves provide a liquidity benefit, measured by the multiplier on the constraint. A bank thus obtains a liquidity benefit from reserves if it draws a sufficiently large shock $\phi_t > \phi^*$. The bank then borrows overnight

$$F_{t+1} = \phi_t \frac{\bar{v}(D_t + L_t)}{1 + \gamma} - M_t.$$  \hspace{1cm} (6)$$

Since $\phi_t \leq \phi^*$, banks can choose a liquidity ratio high enough that they never have to borrow, but can handle even the largest shock just out of reserves.

**Portfolio and capital structure choice**

Banks adjust their portfolio and capital structure subject to leverage costs. They invest in reserves, overnight credit and trees while trading off returns, collateral values and liquidity benefits. They issue payment instruments and adjust equity capital, either through positive dividend payout or negative recapitalizations $y^b_t$. Capital structure choices trade off returns, leverage costs and liquidity costs.

The bank budget constraint says that net payout to shareholders must be financed either through interest on credit lines or through changes in the bank’s positions in reserves, deposits,
overnight credit or trees:

\[
P_t y_t^b = i_{t-1}^L L_t + M_t (1 + i_{t-1}^R) - M_{t+1} - D_t (1 + i_{t-1}^D) + D_{t+1} + (B_t - F_t) (1 + i_{t-1}) - (B_{t+1} - F_{t+1}) + \int_{\Theta_B} ((Q_t^l + P_t x_t) \theta_{t-1} - Q_t^l \theta_t^j) \, dj - e^{\alpha_t c_b (\ell_{t-1}) (\sigma (D_t + L_t) + F_t)} - i_{t-1}^L L_t^b.
\]

In the second line, \( B \geq 0 \) represents a positive position in overnight credit. The last line collects bank leverage costs and credit lines that banks use to pay those costs, both discussed in detail below.

The first line in (7) collects payoff from payment instruments. Interest on reserves \( i_{t-1}^R \) accrues to the bank that held the reserves overnight, regardless of whether those reserves were used by the bank to make a payment. Similarly, deposit interest \( i_{t-1}^D \) is paid by the bank that issued the deposits in the previous period, regardless of whether the deposits were used by endusers to make a payment. Both conventions could be changed without changing the main points of the analysis, but at the cost of more cluttered notation.

In the second line of (7), banks’ tree holdings must come from a subset \( \Theta_B \) of all the trees available to households. One way to think about this restriction is as the result of unmodelled contracting frictions: we could interpret the fruit from trees \( x_t \) as housing services, so the trees represent all claims on housing, which consist of mortgage bonds, as well as – perhaps due to a commitment problem – housing equity. Banks and households both participate in the market for mortgage bonds, whereas only households own housing equity. Alternatively the restriction could be due to regulation, as banks cannot own stocks in some countries.

**Leverage costs**

If the last line in (7) were omitted, the Modigliani-Miller theorem would hold and bank capital structure would be indeterminate. We assume instead that the commitment to make future payments is costly. It takes resources to convince overnight lenders that debt will be repaid, as well as to convince customers that the bank will indeed accept and execute payment instructions. We also assume that convincing lenders and customers is cheaper if the bank owns more assets to back up the commitments, especially if those assets are safe.

To link the cost of commitment depends on banks’ choices, we define the bank leverage ratio

\[
\ell_{t-1} := \frac{\sigma (D_t + L_t) + F_t}{M_t + \rho \int_B Q_t^l \theta_{t-1}^l dj + B_t}.
\]

where \( \sigma \) and \( \rho \) are fixed parameters. A key feature of this ratio is that outstanding credit lines – an off balance sheet commitment – is included in the numerator together with debt. While loan commitments are counted in some leverage concepts, they are often absent from standard accounting measures. Our perspective here is that credit lines are valuable only if they are reliable, and the presence of collateral makes them more credible much like deposits and other debt.

Banks choose the leverage ratio \( \ell_{t-1} \) at date \( t-1 \) through their choice of nominal positions at that date. Banks then have to purchase real resources \( c_b (\ell_{t-1}) (\sigma (D_t + L_t) + F_t)/P_{t-1} \) in the goods market at date \( t \) to support that leverage ratio. The cost function \( c_b \) in (7) is smooth,
nonnegative, strictly increasing and concave in bank leverage. We further assume that \( c_b \) slopes up sufficiently fast that banks always choose \( \ell < 1 \). The weight \( \sigma \in [0, 1] \) allows a distinction between two types of commitments in the numerator, payment instruments and debt.

The denominator of the leverage ratio (8) introduces a collateral value for bank assets: the resources needed to convince customers about future commitments are smaller if the bank owns more assets. The weight \( \rho \) allows a distinction between safe assets (reserves and overnight lending) and trees, which we will later assume to be uncertain. Both the weights and the presence of outstanding credit lines \( L \) in the numerator implies that (8) does not generally correspond to accounting measures of leverage.

Since leverage costs take up real resources, we need to address how banks pay for them. The details of this process are not essential and we choose an approach that simplifies formulas. Resources that support leverage chosen at date \( t-1 \) are purchased by banks in the goods market at the price \( P_t \). In order to pay for those goods, banks must arrange credit lines at other banks\(^5\). Banks thus face an additional liquidity constraint that is analogous to that for households:

\[
e^{\pi_t} c_b (\ell_{t-1}) (\sigma(D_t + L_t) + F_t) \leq \bar{v} L_t^b.
\]

(9)

Here the term \( e^{\pi_t} \) converts nominal debt at date \( t-1 \) dollars into nominal expenditure on goods at date \( t \). As long as the interest rate on credit lines is positive, the constraint then binds in equilibrium: banks arrange for a line that is just large enough to cover the leverage costs that will accrue next period. We use the same velocity parameter \( \bar{v} \) as for the household liquidity constraint – in equilibrium this will lead to a quantity equation that ties the price level to total payment instruments and total output.

**Bank optimal choices**

The bank first order conditions are derived in the appendix. Since the bank problem exhibits constant returns to scale, they only pin down the leverage ratio \( \ell \) and the ratio of reserves to payment instruments \( \phi^* \). As long as those ratios are chosen optimally, banks are indifferent between positions in all assets and liabilities that have effective rates of returns equal to the rate of return on equity \( \delta \). Effective rates of returns take into account not only future payoffs, but also the effects of the asset or liability position on leverage cost and the liquidity constraint.

In particular, the presence of leverage costs leads to a determinate optimal leverage ratio. Given households’ indifference (2) between the payment instruments, all instruments provide a liquidity benefit to endusers and therefore require a lower pecuniary benefit. From the bank side, providing payment instruments thus taps a source of funds that is "cheap" in pecuniary terms: both issuing deposits and issuing equity together with extending credit lines is cheaper than just issuing equity. For small leverage, it thus always makes sense to provide payment instruments. Banks increase provision until the marginal leverage is high enough that further leverage is not profitable.\(^6\)

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\(^5\)Equivalently we could assume that banks must hold deposits at other banks.

\(^6\)In effect, the marginal cost of debt is a smoothed-out version of what it would be in a model with a collateral constraint: if we assumed e.g. \( L \leq \alpha K \) for some number \( \alpha \), the extra cost of debt would be zero up to the constraint and infinite thereafter.
2.3 Government & equilibrium

We treat the government as a single entity that comprises the central bank and the fiscal authority. The government issues reserves $M_t$, borrows $B_t^g$ in the overnight market and chooses the reserve rate $i_t^R$. The government also makes lump sum net transfers to households so that its budget constraint is satisfied every period. Below we further consider particular policies that target endogenous variables such as the overnight interest rate. Such policies are still implemented using the basic tools $M_t$, $B_t^g$ and $i_t^R$.

Just like financial firms, the government incurs a cost of issuing debt, above and beyond the pecuniary cost. The government differs from firms in that it has the power to tax and hence the (implicit) collateral that is available to it. We define government leverage as $\ell^G_t = (M_{t+1} + B_{t+1}^g) / P_{t+1} \Omega_{t+1}$ and denote the date $t$ government leverage cost as $e^{\tau_t} c_g (\ell^G_{t-1}) (M_{t+1} + B_{t+1}^g)$ where $c_g$ is increasing and convex, as is the bank leverage cost function $c_b$. The more real debt $(M_t + B_t^g) / P_t$ the government issues relative to the labor income tax base $\Omega_t$, the more resources it must spend to convince lenders – so far, only banks – that it will repay. In order to pay leverage cost, the government is required to arrange a credit line from banks, denoted $L_t^g$.

**Market clearing**

Equilibrium requires that markets clear at the optimal choices of banks and households, taking into account government policy. Tree market clearing requires that banks or households hold all trees. The overnight credit market clears if borrowing by banks $F_t$ plus government borrowing $B_t^g$ equals aggregate bank lending $B_t$. Banks must hold all reserves. Since the cross sectional distribution of bank portfolios is indeterminate, we now use the symbols $M_t$, $L_t$, $D_t$ etc to denote aggregate bank positions.

The goods market clears if households consume the endowment and all fruit from trees, net of any resources spent by banks and the government as leverage costs. We denote the total quantity of goods sold at date $t$ by $T_t$. In nominal terms, goods market clearing means

$$P_t T_t = P_t C_t + e^{\pi_t} c_g (\ell^G_{t-1}) M_t + e^{\pi_t} c_b (\ell_{t-1}) (\sigma (D_t + L_t) + F_t).$$

Below we will assume that $T_t = \Omega_t + x_t$ is constant over time and only the composition of transactions is determined in equilibrium. For example if banks and the government are more levered, then consumption must be lower.

The market for payment instruments clears if deposits and credit lines offered by banks equal payment instruments demanded by households plus credit lines arranged by banks and the government. The model does not introduce a functional difference between credit lines and deposits. To emphasize their similarity, we focus on equilibria with $i_t^D = \delta_t - (i_t^D - \pi_{t+1})$ and treat the two types interchangeably. Market clearing in the market for payment instruments requires that $D_t^h + L_t^h + L_t^b + L_t^g = D_t + L_t$. If the three goods market liquidity constraints (for households, banks and the government) all bind, the total real quantity of payment instruments satisfies

$$P_t T_t = \bar{v} (D_t + L_t).$$ (10)

In order to for society to handle the transactions $T_t$, banks must supply a positive amount of payment instruments in real terms. Given a finite real value of amount of collateral, banks thus
incur leverage costs. As a result, payment instruments are costly to endusers and endusers’ liquidity constraints bind: (10) holds with equality and works like a quantity equation that relates the price level to nominal supply of payment instruments.

While enduser liquidity constraints always bind, banks’ liquidity constraints may or may not bind, depending on how many real reserves are available relative to transactions $T_t$ as well as other collateral. We say that reserves are scarce at date $t$ if $\phi^*_t < \bar{\phi}$, so the bank liquidity constraint binds with positive probability at date $t + 1$. In contrast, reserves are abundant if $\phi^*_t \geq \bar{\phi}$ so banks are sure that the constraint will not bind. In principle, reserves could be scarce at date $t$ even though some banks are constrained at date $t$ itself.

The appendix derives a system of equations that characterize equilibrium. It describes the dynamics of the endogenous prices – the interest rates $i_t$ and $i^L_t$, the price of bank trees $Q^t_1$, the nominal price level $P_t$ – and three variables on that describe bank balance sheets – aggregate payment instruments $D_t + L_t$ as well as the liquidity ratio $\phi^*_t$ and the leverage ratio $\ell_t$, both of which are equal across banks in equilibrium. The appendix further derives an approximation to the system that works as long as rates of return are small numbers, which can be guaranteed by assumptions on exogenous policy parameters. The following sections then study the steady state of that system for different exogenous parameters.

**Neutrality of nominal government liabilities**

Government policy determines the total amount of nominal liabilities $M_t + B_t^g$ as well as their composition. If the path of both reserves and government debt are increased by the same factor, then the path of the nominal price level $P_t$ is also increased by that factor. Mechanically, reserves and government debt appear in the equations characterizing equilibrium only in the form of real reserves $m_t = M_t / P_t$ and real borrowing $B_t / P_t$. This is because our model assumes flexible prices and no nominal rigidities in the private sector.

The neutrality property here differs from that in cash-in-advance models in which reserves (or currency) directly provide liquidity services to households. In a standard model, an increase in the money supply along increases the price level in the same proportion – the outstanding amount of nominal government debt is not important. This is not true in our model because banks can use government debt as an additional nominal collateral used to produce nominal payment instruments.\(^7\)

### 3 Steady state equilibrium

We now consider steady state equilibria with constant output $\Omega + x$ and constant rates of return. We thus restrict attention to policies that imply constant growth rates for the nominal quantities $M_t$ and $B_t^g$ so that the $B_t^g / M_t$ is also constant. With constant rates of return, the marginal rate of substitution of wealth across dates equals the discount rate, that is $\bar{\delta}_t = \delta$. Moreover, the key ratios chosen by banks, leverage $\ell$ and the liquidity ratio $\phi^*$, are constant over time. With fixed output, payment instruments and the price level also grow at same rate

\(^7\)The special role of government thus comes from the endogeneity of deposits and their exclusive use as a medium of exchange. It does not come from "non-Ricardian" fiscal policy. In our model, government surplus is not exogenous but instead adjust so as to satisfy the government budget constraint.
as nominal government liabilities. The nominal price level and the real quantity of reserves are endogenous and depend on how many payment instruments banks produce for a given level of reserves.

3.1 Graphical analysis

The predictions of the model can be characterized by reducing the system of equations characterizing equilibrium to only two equations in two key variables: bank leverage $\ell$ and the real quantity of reserves $m$. One equation – the capital structure curve – describes the amount of leverage that banks require in order to handle transactions $T$ for given real reserves $m$. It depends on policy because the government can change the mix of collateral available to banks.

The second equation – the liquidity management curve – describes liquidity management; it depends on policy because the government can change the opportunity costs of holding reserves. Properties of equilibrium can be studied by plotting the two curves in the $(m, \ell)$-plane, as in Figure 1. Equilibrium is described by their intersection and its welfare properties can also be studied graphically since the welfare costs of leverage can be written as functions of $m$ and $\ell$.

A figure in $(m, \ell)$-plane helps think about not only real reserves and leverage, but also a number of other variables that are related in simple ways to those ratios in equilibrium. In particular, location of the equilibrium along the vertical axis provides insight on real asset

![Figure 1: The green line is the capital structure curve. The blue line is the liquidity management curve. In the yellow shaded region, reserves are abundant, $m \geq \bar{m}$.](image-url)
values, whereas location along the horizontal axis contains information about bank balance sheets and the price level. We now explain these relationships. Derivations of all equations are collected in the appendix.

The scarcity of reserves and the overnight credit market

The \((m, \ell)\)-plane splits into two halves that correspond to scarce and abundant reserves, respectively. Indeed, in a steady state with constant transactions \(T\), real payment instruments are also constant and banks’ liquidity ratio is proportional to real reserves:

\[ \phi^* = \frac{1 + \gamma}{T} m. \]  

(11)

The vertical pink line in Figure 1 marks the critical quantity of real reserves \(\bar{m} = \frac{T}{1 + \gamma} \phi\) such that reserves are abundant if \(m \geq \bar{m}\) (the yellow region shown to the right) and reserves are scarce if \(m < \bar{m}\).

The steady state real amount of overnight credit is also linked directly to transactions and reserves. Bank borrowing is found by integrating over all banks that receive a liquidity shock beyond \(\phi^*\):

\[ \frac{F_t}{P_t} = \frac{T}{1 + \gamma} \int_{1+\gamma \phi}^{\phi^*} \left( \phi - \frac{1 + \gamma}{T} m \right) dG(\phi) =: f(m; \gamma), \]

The function \(f\) is decreasing in both arguments. Indeed, if the banking system has more real reserves \(m\) to handle a given amount of transactions, then less overnight credit is needed. Overnight borrowing by banks can therefore be read on the horizontal axis going left – it is zero when reserves are abundant. Moreover, if more transactions can be netted within the day (higher \(\gamma\)), then banks again run out of reserves less often and borrow less overnight.

Asset market participation and intermediary asset pricing

Except for the restriction that reserves must be held by banks, we have not made assumptions on asset market segmentation. Instead, participation patterns of intermediaries and households are determined endogenously in equilibrium. Throughout, the basic principle at work is familiar from other models with short sale constraints: assets are held – and hence priced – by the investor who likes them the most, either because that investor derives nonpecuniary (liquidity or collateral) benefits from them or later also because that investor has more optimistic beliefs.

Consider participation in overnight credit and tree markets. Since the production of payment instruments entails leverage costs, banks obtain a positive collateral benefit from assets they are eligible to hold. As a result, households do not invest in any asset markets that banks can invest in: banks bid up the price of any asset accessible to banks until its return is below the discount rate and the asset is unattractive to households. For overnight credit and bank trees, banks are thus the only marginal investors.

The real overnight interest rate satisfies the bank Euler equation

\[ \delta - (i - \pi) = \kappa(\ell). \]

(12a)

where \(\kappa(\ell) = c'_\ell(\ell) \ell^2\) is the marginal benefit of collateral which is increasing in leverage. The condition resembles other "intermediary asset pricing" equations in the literature, as it draws
a connection between returns and leverage. Importantly, our concept of leverage is special and reflects the provision of payment instruments as opposed to book or market leverage in an accounting sense.

The real interest rate is inversely related to leverage; in figure 1, it can be read on the vertical axis going down. Intuitively, more levered banks are willing to pay more for collateral and bid up prices, thus lowering returns. The same logic applies to bank trees, which are priced by a similar Euler equation. In steady state, cash flows on bank trees, denoted $x_B$ are discounted at a low rate that accounts for the collateral benefits: their steady state value is $x_B/\left(\delta - \rho \kappa (\ell)\right)$. In the figure, it can be read on the vertical axis going up.

**Payment instruments and the nominal price level**

Location along the horizontal axis contains information about payment instruments and the price level. The "money multiplier" $(D + L)/M = \bar{v}T/m$ is inversely proportional to $m$ and can be read on the horizontal axis moving left. The nominal price level is determined by nominal government liabilities. It is helpful to write the relationship between the price level and real reserves as

$$P_0 = \frac{M_0 + B_0^g}{m(1 + B_0^g/M_0)}.$$  

For given nominal liabilities $M_0 + B_0^g$, we can read off the the price level on the horizontal axis moving left as long as the split of government debt into reserves and bonds $B_0^g/M_0$ does not change. For comparative statics that change that mix, for example when we look at the result of open market operations, the price level no longer changes one for one with $\delta$, but the figure is still helpful to see the effect on the price level, as explained below.

**The capital structure curve**

How many real reserves $m$ are required in order for banks to manage transactions $T$ with given bank leverage $\ell$? Steady state real reserves and leverage are linked by

$$\ell = \frac{\sigma T/\bar{v} + f(m; \gamma)}{m(1 + B_t^g/M_t) + \rho \delta - \rho \kappa (\ell)} + f(m; \gamma).$$  

We refer to pairs $(m, \ell)$ that satisfy this relationship it as the capital structure curve. Since $B_t^g/M_t$ is exogenous, the curve also tells us how much government leverage is required to handle transactions with given bank leverage.

The capital structure curve slopes down in the $(m, \ell)$-plane, as shown as green line in Figure 1. The basic intuition is that the real quantity of payment instruments $\sigma T/\bar{v}$ is pinned down by the volume of transactions, so lower leverage requires more collateral. Since reserves contribute to collateral, we have that – other things equal – lower leverage requires more reserves. As a stark example, consider a "narrow bank" which holds no trees or overnight credit – its only collateral is reserves or short term government debt. The leverage ratio is then simply the ratio of payment instruments to reserves: $\ell = \sigma T/\bar{v}(1 + B_t^g/M_t)m$. For a narrow bank, a downward sloping convex capital structure curve thus follows directly from the definition of leverage.

In general, two more subtle effects further contribute to a downward slope. First, banks with more reserves (and hence higher liquidity ratios) run out of reserves less often, which results in lower outstanding interbank credit and hence lower leverage. Indeed, since every dollar of
interbank credit is both an asset and a liability to the banking sector, a reduction in interbank credit also reduces overall leverage. Second, if leverage is lower, banks compete less for trees, so tree prices and hence the value of collateral value fall. As a result, lower leverage requires even more reserves.

The capital structure curve is closer to the origin if banks have access to more collateral other than reserves. Indeed, an increase in the ratio of government bonds to reserves $B^g_t/M_t$ shifts the curve to the left: if more government bonds are available to back payment instruments, then banks require fewer real reserves to achieve any given leverage ratio. An increase in the cash flow from bank trees $x_B$ has the same effect: if more private sector assets are available to banks, then fewer reserves are required. Both shifts also entail an increase in interbank credit for given leverage ratio: this increase – which contributes to leverage – is feasible because of the increase in collateral.

The capital structure curve is steeper if banks’ portfolio share of nominal collateral is larger. Consider in particular the region with abundant reserves, where reserves and government bonds are the only nominal assets. If a larger share of banks’ collateral consists of reserves as opposed to trees, then any given change in $m$ implies a larger change in leverage, that is, we have a steeper capital structure curve. Intuitively, an increase in $m$ corresponds to an decrease in the nominal price level and an increase in the value of money. The more nominal collateral banks hold relative to trees, the more an increase in the value of money increases the value of collateral and the more leverage becomes possible.

The location of the capital structure curve also depends on velocity as well as interbank netting arrangements. For given real reserves, an economy with higher velocity requires lower bank leverage – this is because fewer payment instruments are needed to handle a given amount of transactions. Similarly, for given real reserves, an economy with more intraday netting (higher $\gamma$) requires lower leverage. This because that economy requires less overnight credit. This effect is limited to the scarce reserves region – if reserves are abundant then netting is irrelevant for leverage.

The liquidity management curve

Optimal liquidity management equates the opportunity cost of reserves to their collateral and liquidity benefits:

$$
\delta - (i^R - \pi) = \kappa(\ell) + (1 - G(m(1 + \gamma)/T))(\lambda(\ell) - \kappa(\ell)),
$$

where $\lambda(\ell) = c_b(\ell) + c^*_b(\ell)\ell$ is the marginal cost of leverage. For a given real return on reserves, this equation represents pairs $(m, \ell)$ such that banks optimally choose their liquidity ratio – and hence real reserves $m$ – given leverage $\ell$. We refer to it as the liquidity management curve.

The left hand side of (14) is the opportunity cost to banks of holding reserves, relative to shrinking the balance sheet and paying back shareholders who demand the return on equity $\delta$. In equilibrium, it must equal the benefit of reserves as collateral as well as for liquidity purposes. The collateral benefit is simply $\kappa(\ell)$; it is increasing in leverage. The second term on the right hand side is the liquidity benefit: with probability $1 - G(\phi^*)$, the bank runs out of reserves and obtains the benefit $\lambda - \kappa$.

The liquidity benefit of reserves is decreasing in the real quantity of reserves – in fact, once reserves are abundant for $m \geq \bar{m}$ the liquidity benefit shrinks to zero. As long as reserves
are scarce, the liquidity benefit of reserves $\lambda - \kappa$ is positive and increasing in leverage. In equilibrium, every dollar of interbank credit entails leverage cost for the borrowing bank but adds collateral benefits for the lending bank. Since the former effect is larger, having to borrow overnight implies a penalty for running out of reserves that increases with leverage.

Putting together these effects, the liquidity management curve (14) consists of two pieces, as shown in Figure 1. As long as reserves are scarce, it describes an upward sloping curve in the $(m, \ell)$ plane: more reserves lower the liquidity benefit, and are consistent with the same opportunity cost only if leverage is overall higher. Once reserves are abundant, the liquidity management curve is flat: the economy is at a leverage ratio such that the opportunity cost of reserves is just compensated by the collateral benefit alone.

The location of the liquidity management curve depends on the real return on reserves $i^R - \pi$. In particular, a lower return on reserves shifts the curve up, that is, banks choose higher leverage for a given quantity of real reserves. Intuitively, high opportunity costs of holding reserves make it expensive for banks to handle transactions. As a consequence, banks try to economize on reserves. Keeping the same real quantity of reserves is profitable only at higher levels of leverage.

The location of the liquidity management curve also depends on interbank netting arrangements, at least as long as reserves are scarce. Greater "netting efficiency" (higher $\gamma$) shifts the curve to the left: if intraday credit is more effective at making payments, fewer reserves are needed to handle them. A leftward shift implies that the curve remains unchanged in the abundant reserves region where netting is not important. At the same time, the mix of collateral available in the economy – for example how many trees are available relative to government liabilities – is not relevant for liquidity management and does not affect the location of the curve.

### 3.2 Equilibrium & changes in policy

Equilibrium real reserves and leverage are determined by the intersection of the capital structure and liquidity management curves. Whether reserves are scarce or abundant in equilibrium is determined by the interaction of policy and the availability of collateral. In particular, the government can set the real rate on reserves so as to select its preferred region. For example, increasing the real rate on reserves shifts the liquidity management curve down towards an equilibrium with abundant reserves.

At the same time, holding fixed policy, an equilibrium with abundant reserves obtains when collateral is relatively scarce: as discussed above, a decrease in the cash flow of trees accessible to banks $x_B$ or a decline in the relative amount of government bonds $B^g_t / M_t$ shifts the capital structure curve up, and reserves become relatively more abundant.

We now consider comparative statics of steady states. The time period in the model should be thought of as very short, such as a day. We are therefore comfortable using the model for thinking about the behavior of asset prices, payments and credit over a sequence of years, such as the recent boom bust cycle. In particular, we are interested in the effect of shocks to agents’ belief about future asset payoffs (such as those on claims to housing) as well as monetary policy responses and study those effects as a sequence of comparative statics.
Monetary/fiscal policy & interest on reserves

Since the steady state rate of inflation equals the growth rate of reserves, a government that commits to a growth rate of nominal liabilities effectively controls the real rate on reserves. This specification helps think about two policy regimes. One is traditional: the government sets debt and reserves while interest on reserves is zero. The other – more recently popular – regime is one where the central bank has issued so many reserves that they are abundant, and the spread between $i$ and $i^R$ is zero. The central bank then makes policy by moving around the interest rate on reserves which can be positive or negative.

Consider how changes in monetary policy affect equilibrium in the two regions. First, suppose we start from an initial equilibrium with $i^R = 0$ and scarce reserves – the typical situation in many countries before the recent financial crisis. Suppose now the government decides on faster growth of nominal liabilities. Mechanically, the capital structure curve remains unchanged, whereas the liquidity management curve shifts up. Figure 2 shows the shifted curve as dotted line.

Banks optimally respond to higher inflation (and hence higher opportunity costs of holding reserves) by choosing higher leverage for any given level of real reserves. They provide more nominal payment instruments relative to reserves (the "money multiplier" increases) and the price level in the new steady state is higher. As reserves are more scarce in real terms, overnight credit also increases. At the same time, more leveraged banks compete more for collateral and bid up asset prices: the price of bank trees increases and the real overnight interest rate declines.

Figure 2: Faster growth rate of nominal liabilities when reserves are scarce
In comparison, consider increasing the reserve rate when reserves are already abundant, as the Fed did in December 2015. Assume further that the government continues to commit to the same constant growth rate $\pi$ of nominal liabilities. The mechanical effect is the opposite of the above: as the real rate on reserves increases, the liquidity management curve shifts down. As the opportunity cost of holding reserves falls, banks find it profitable to handle the same payments with lower leverage. They produce fewer payment instruments relative to reserves, the money multiplier shrinks and the nominal price level in the new steady state is lower. Since reserves are already abundant and now become even more attractive, the overnight credit market remains inactive. Banks also require less collateral so that asset prices decline.

The two examples show that changes to the opportunity costs of holding reserves have intuitive effects. In particular, expansionary policy (such as faster money growth) lowers interest rates and increases the price level, whereas contractionary policy (such as a higher nominal rate on reserves) does the opposite. We emphasize that the effects are permanent — they result from comparative statics across steady states. This is in contrast to many models with sticky prices or segmented markets, where "liquidity effects" on the real interest rate are temporary phenomena. Permanent effects arise in our model because the opportunity costs of holding reserves lead banks to change the way they produce payment instruments, with effects on the cost of leverage and the value of collateral.

*Open market operations & the collateral mix*

By engaging in open market policy — for example, a swap of reserves for government bonds —
the government can alter the tradeoff between bank leverage and government leverage required to handle transactions. In other words, it shifts the capital structure curve. This is different from changes to the real return on reserves that alters the tradeoffs in liquidity management. To illustrate, consider a comparative static that increases reserves and offsets this change by an equal change in bonds: we move from an initial equilibrium with \((B^g_0, M_0)\) to a new equilibrium with \((\tilde{B}^g_t, \tilde{M}_t)\), where \(\tilde{M}_0 - M_0 = B^g_0 - \tilde{B}^g_0 > 0\). As before, reserves and bonds then grow at the same rate \(\pi\) throughout.

The expansionary open market operation can be summarized by a decrease in the ratio \(B_t/M_t\) which shifts the capital structure curve to the right. If reserves are abundant in the initial equilibrium, then the policy has no effects: it does not change nominal liabilities so the price level remains unchanged, as does leverage and interest rates. The horizontal shift in the curve thus reflects the change nominal reserves \(\tilde{M}_0 - M_0\).

In contrast, when reserves are scarce as in Figure 4, then a purchase of bonds for reserves permanently increases the price level and lowers the real interest rate. Mechanically, since the liquidity management curve slopes up, the change in real reserves is smaller than \(\tilde{M}_0 - M_0\), indicating an increase in the price level. Intuitively, if there are fewer bonds outstanding for any given level of real reserves, banks have less collateral and higher leverage. They produce more payment instruments relative to reserves and the nominal price level increases. At the same time, collateral is less scarce and the real interest rate increases, while real asset prices decline.

![Figure 4: Open market operations when reserves are scarce](image-url)
3.3 The role of the nominal interest rate

Some central banks conduct monetary policy by following a nominal interest rate rule. In practice, the rule is typically implemented by open market policy. For example, during the scarce reserves regime in place in the US until 2008, the New York Fed’s trading desk bought and sold bonds of various maturities in exchange for reserves in order to move the overnight interest rate (the Federal Funds rate) close to the Fed’s target. More recently as reserves have become abundant the Fed Funds rate and the interest rate on reserves have been essentially the same, and the policy lever is the interest rate on reserves. It is then tempting to simply transfer existing analysis of interest rate rules to the abundant reserves environment even though the policy implementation is different.

In many monetary models, the details of how the central bank implements the interest rate rule are indeed irrelevant – the nominal interest rate alone summarizes the stance of monetary policy. In particular, many models use households’ optimal choice between currency and short term bonds to derive optimal real balances as a function of the nominal interest rates and consumption. At the same time, intertemporal asset pricing equations – and possibly price setting equations – imply a path for inflation. The path for the money supply can then be inferred ex post so as to generate the implied path for real balances, but is often omitted from the analysis altogether. In particular, it does not matter whether policy is implemented via open market purchases or interest on reserves.

In our model, policy cannot be summarized by the nominal interest rate alone. As discussed above, policy matters in two ways. On the one hand, it can change the nominal rate on reserves. On the other hand, it can change the collateral mix between reserves and government bonds, which matters as long as reserves are scarce. Both policy actions affect the nominal interest rate. First, with scarce reserves, the same nominal interest rate can be achieved with many combinations of interest on reserves and open market purchases that have different implications for real interest rates, inflation and real reserves. Second, with abundant reserves when open market purchases are irrelevant, interest on reserves is the key policy tool.

Interest rate policy with scarce reserves

Consider first the case of scarce reserves. We start from an initial equilibrium in that region; it is generated by initial parameters $i^R, \pi$ and $B_0/M_0$ and implies some initial overnight rate. Holding fixed $i^R$, we now choose a new target overnight rate that is above the reserve rate and ask how $\pi$ and $B_0/M_0$ can change to implement it. To proceed graphically, we trace out all equilibrium pairs $(m, \ell)$ that are consistent with the spread $i - i^R$:

$$i - i^R = (1 - G (m (1 + \gamma) / T)) (\lambda (\ell) - \kappa (\ell)) .$$  \hspace{1cm} (15)

The spread is the opportunity cost of holding reserves rather than lending overnight. It must be equal to the liquidity benefit of reserves on the right hand side since the collateral benefits of the two assets are the same.

The key properties of the curve in $(\ell, m)$ plane described by equation (15) are illustrated in Figure 5. First, the red curve is upward sloping: if banks are more leveraged, then overnight credit entails higher leverage cost; in order to maintain the same opportunity cost of reserves there must be more real reserves. Second, the curve never enters the abundant reserves region. As reserves become more abundant, leverage must rise to maintain a positive spread. Third,
the new curve lies below the liquidity management curve at the initial equilibrium $m$. In order for the new overnight rate to be lower requires a lower liquidity benefit, which requires lower leverage for given reserves $m$. Finally, the curve is independent of $\pi$ and $B_0/M_0$, the two parameters describing policy.

The equilibrium pair $(m, \ell)$ not only satisfies (15), but must also lie on the capital structure and liquidity management curves. How can the government change policy to move to the new lower overnight rate? There are two stark options. The first option is for the government to announce a lower growth rate of nominal liabilities $\ddot{\ell}$, shifting the liquidity management curve down until all three curves intersect. This policy leaves the collateral mix unchanged but only lowers the opportunity cost of reserves. The second option is for the government to engage in expansionary open market operations, shifting the capital structure curve to the right until all three curves intersect. This policy leaves the opportunity costs on reserves unchanged but changes the collateral mix.

The two extreme policies produce the same change in the overnight nominal rate. At the same time, they have very different implications for the real interest rate and inflation, as well as for real reserves and overnight credit. In the first option, lower money growth reduces inflation which contributes to the decline in the nominal interest rate. The effect is less than one-for-one however – the equilibrium real interest rate actually increases. The reason is that lower opportunity costs of reserves reduce bank leverage and lower the collateral benefit of overnight lending. In the other option, open market purchases leave inflation unchanged. However, fewer bonds implies lower collateral and higher bank leverage.
In addition to the two extreme policies just sketched, many other policies are also consistent with the new target nominal overnight rate. Indeed, we can combine open market purchases with announcement of future growth of liabilities: we then shift both curves at once, rather than one at a time as for the extreme policies. The only requirement on the shifts is that the new equilibrium ends up on the curve described by equation (15). In particular, the same nominal interest rate is compatible with an entire range of equilibrium "real balances" $m$.

A key difference between our model and other models of scarce outside money is that the only medium of exchange for endusers is payment instruments produced by banks. Outside money – here reserves – is only one input into the production of payment instruments. In particular banks also use government bonds as collateral to back payment instruments. As a result, the spread between the overnight rate and the reserve rate measures the scarcity of reserves for banks; it does not measure the scarcity of payment instruments in the economy as a whole. In particular, there is not a unique amount of reserves implied by a given volume of transactions and a spread.

**Interest rate policy with abundant reserves**

Consider policy in the abundant reserve regime. Can the government describe policy only by the single interest rate $i = i^R$? Equilibrium is described by (12a) and (13), which determine $m$ and $\ell$ for a given real return on reserves. As a result, a nominal reserve rate alone cannot pin down real reserves and bank leverage. Similarly, a feedback rule that relates the rate on reserve to inflation, for example $i = g(\pi)$, does not uniquely determine $m$, $\ell$, inflation and the real interest rate. This is true even if we directly select a rule for the real rate as a function of $\pi$, thus eliminating possible multiplicity coming from the shape of $g$ that has been discussed in the literature.

Our model differs from other monetary models in what happens once outside money becomes abundant. Consider first models with bonds and currency only. At the zero lower bound in such models, bonds and outside money become perfect substitutes to endusers, so the medium of exchange (currency) loses its liquidity benefit. In the current model, endusers hold neither bonds nor outside money – both are held only by banks. Equating $i$ and $i^R$ makes bonds and outside money perfect substitutes for banks, but does not remove the liquidity benefit of the medium of exchange, namely payment instruments produced by banks.

There are also models in which reserves, bonds and currency coexist. In such models, $i = i^R > 0$ makes bonds and reserves perfect substitutes. At the same time, currency remains a scarce medium of exchange that is relevant for some transactions. The reserve rate represents the spread between reserves and currency; it measures the scarcity of currency and related the demand for real balances to real variables such as consumption. The tradeoff between currency and reserves is what makes enables those models to work with interest rate rules in the usual way even when reserves are abundant.

### 3.4 Optimal policy

The optimal payments system minimizes the loss of resources due to leverage. Our technological assumptions say that a given volume of transactions requires a fixed amount of payment instruments supplied by banks, as well as some outside money supplied by the government that
can in turn serve as collateral for banks. Total consumption lost every period in steady state can expressed as a function of leverage and liquidity:

\[ c_g \left( m(1 + B_t/M_t)/\Omega \right) m(1 + B_t/M_t) + c_b (\ell) (\sigma T/\bar{v} + f(m; \gamma)). \]  

(16)

Provided that the leverage cost of the government slopes up fast enough, the indifference curves are downward sloping and concave, as shown in Figure 6.

We consider the best steady state equilibrium that the government can select by choice of its two policy instruments, the collateral mix represented by \( B_t/M_t \) and the real return on reserves \( i_R - \pi \), which determines the tax on reserves. The optimal policy problem is to choose those instruments together with \( m \) and \( \ell \) to minimize (16) subject to the capital structure curve (13) and the liquidity management curve (14).

If the government can freely choose the ratio of bonds to reserves, it is optimal to set \( B_t/M_t = 0 \). Indeed, while bonds and reserves provide the same collateral services, reserves also provide liquidity services, which lowers the need for interbank borrowing and hence costly bank leverage. Since the model focuses on the provision of payment services, there is no benefit of government bonds per se, nothing is lost by just issuing reserves. More generally, the way fiscal policy is conducted independently of monetary policy may imply that there is a constraints on \( B_t/M_t \). We can then view the welfare cost as as a function in \( \ell \) and \( m \) with \( B_t/M_t \) a fixed parameter.\(^8\)

\(^8\) Alternatively, we could capture fiscal policy by a given real amount of bonds, say \( b \). The welfare cost can
The return on reserves directly affects neither the welfare cost nor the capital structure curve. We can therefore find the optimal solution in two steps. We first find a point \((m, \ell)\) on the capital structure curve (for given \(B_t/M_t\)) that minimizes (16). The optimal real return on reserves is then whatever return shifts the liquidity management curve so that the equilibrium occurs precisely at that optimal point. If the indifference curves are concave and the capital structure curve is convex – which is a reasonable assumption if the effects of interbank credit are relatively small – then we obtain an interior solution as shown in Figure 6.

Should reserves be abundant? The figure suggests that this is not necessarily the case. Indeed, if the government leverage cost curve slopes upward very steeply, then it may be optimal to run a system with scarce reserves, in which real government is much lower than the debt required to run the payments system. It is better to have banks rely on other collateral in order to back payment instruments. However, if the government can borrow cheaply at will, so that its leverage cost is close to zero, then it makes sense to move towards narrow banking where reserves make up the lion’s share of bank portfolios.

### 4 Securities markets and the payment system

In this section we consider the interplay between securities markets and the payment system. We maintain throughout our focus on steady states. We first introduce uncertainty premia, the key source of fluctuations in asset prices. This allows to discuss the effect of uncertainty shocks on the supply of payment instruments as well as unconventional monetary policy – the government purchases trees that carry uncertainty premia. These questions can be studied even if the only link between tree (that is, securities) markets and the payments system is that banks invest in trees.

We then extend the model to introduce two additional links: banks lend short overnight to institutional investors and institutional investors use payment instruments to trade assets. To clarify the effect of each link in isolation, we introduce two types of asset management firms: carry traders buy trees on margin, whereas active traders face liquidity constraints for some asset purchases. Both types of firms otherwise work like banks: they are competitive firms owned by households that have access to a subset of trees and maximize shareholder value.

#### 4.1 Introducing uncertainty and collateral quality

We capture a change in uncertainty as a change in beliefs about the output: we assume that households behave as if output will fall at a rate \(s\). At the same time, actual output remains constant at \(\Omega + x\) throughout. One way to think about these beliefs is that households are simply pessimistic. Our preferred interpretation is ambiguity aversion: households contemplate a range of models for payoffs, and evaluate consumption plans using the worst case model. In either case, the key effect of pessimistic valuation is to generate premia on assets: an observer (such as an econometrician measuring the equity premium) will observe low prices relative to

\[
\text{then be written with total debt equal to } m + b \text{ as opposed to } m (1 + B_t/M_t). \text{ The basic tradeoff remains the same.}
\]
payoffs and hence high average returns.

To define equilibrium, we must take a stand not only on beliefs about exogenous variables, but also about endogenous variables such as the nominal price level and asset prices. We follow Ilut and Schneider (2014) who also capture the presence of uncertainty via low subjective mean beliefs about exogenous variables: beliefs about endogenous variables follow from agents’ knowledge of the structure of the economy. In particular, agents know the policy rule of the government and that banks maximize shareholder value given the households’ discount factor. Households’ worst case beliefs thus also affect bank decisions; shareholder value (3) is replaced by its worst case expectation.

Characterizing equilibrium is particularly simple if households expect the government to run a monetary policy that implements a constant inflation rate \( \pi \). Households therefore perceive ambiguity only about output and hence the number of transactions \( T \), but not about the evolution of the price level. They do perceive ambiguity about the path of reserves which differs from the actual constant growth path; the common denominator between the perceived and the actual policy is only that they keep inflation constant at \( \pi \). This assumption serves to focus on changes in uncertainty in asset markets only.

Working with mean beliefs allows us to capture the essential features of uncertainty with minimal changes to the model equations. The household first order condition relates the price households observe in the market in the current period with their expected future price. Since households behave as if they live in a steady state in which tree payoffs fall at the rate \( s \), the first order condition is now

\[
\frac{Q_j}{P} \geq e^{-\delta-s} \left( \frac{Q_j}{P} + x \right).
\]

In other words, payoffs on trees held by households are now discounted at the rate \( \delta + s \).

To see how ambiguity generates premia on assets, consider a tree \( j \) held by households so that (17) holds with equality. The equilibrium price of the tree is \( x/(\delta + s) \). An econometrician who observes prices and payoffs measures a return \( \delta + s \) which is higher than the discount rate. If there was also a second "safe" tree held by households that earns exactly the discount rate, the econometrician would measure an equity premium on the uncertain tree. In terms of comparative statics, an increase in uncertainty captured by an increase in \( s \) leads to higher premia and lower prices.

**Uncertainty and collateral quality**

It is natural to assume that trees that are more uncertain also represent worse collateral. In this section, we make the weight that trees receive in the aggregation of collateral explicitly a decreasing function \( \rho(s) \) of the spread. A change in uncertainty thus has two effects on banks tree portfolios. There is a direct effect on prices. Moreover, the fact that the trees are uncertain makes them worse collateral per dollar of funds invested in them.

The bank first order condition for trees accessible by banks is now

\[
\delta = r_j - s + \rho(s) \kappa(\ell).
\]

The presence of uncertainty does not change the fact that all accessible trees are held within banks in equilibrium since trees still provide collateral benefits. Returns on assets held by banks
are affected by two opposing forces: compensation for uncertainty born by shareholders tends to make returns higher, while the collateral benefit tends to make returns lower. The presence of uncertainty thus also puts an additional wedge between the return on trees held by banks and the return on safer overnight credit.

In terms of our graphical analysis, the only change is to the value of bank trees in the capital structure curve. In particular, the first term in the denominator on the right hand side of (13) should be replaced by

\[ \frac{x_B}{\delta + s - \rho(s) \kappa(\ell)} \]

the value of bank trees after discounting at the higher rate that includes the uncertainty premium. Holding fixed leverage, an increase in the uncertainty premium \( s \) lowers the value of bank trees for given leverage and the value of banks’ collateral. For leverage to remain the same, the value of reserves must increase, that is, price level falls. It follows that the capital structure curve shifts to the right, as in Figure 7. The liquidity management curve is not affected by changes in \( s \).

4.2 An increase in uncertainty and policy responses

The value and collateralizability of trees affects the scarcity of reserves even if policy (described by \( i_R - \pi \) and \( B_t/M_t \)) does not change. Indeed, starting from an equilibrium with scarce
reserves, an increase in uncertainty lowers the real interest rate and the price level. This is because the drop in collateral values makes it more expensive for banks to create payment instruments. As a result, the supply of payment instruments declines and generates deflation. In the new equilibrium, bank leverage is higher, the bank liquidity ratios are higher and the ratio of interbank credit to payment instruments is lower.

If the increase in uncertainty is large enough, the capital structure curve shifts so far to the right as to push the economy into the abundant reserves region, as shown in Figure 7. At this point, further increases in uncertainty no longer change the overnight interest rate. The real quantity of reserves is now so high relative to the real quantity of transactions that all liquidity shocks can be handled with reserves and intraday credit alone. The interbank overnight market shuts down entirely. Further increases in uncertainty do still lower the value of collateral, the nominal quantity of payment instruments and the price level.

An increase in uncertainty is an attractive candidate for a shock that could have occurred at the beginning of the recent financial crisis. It is consistent with an increase in asset premia, a drop in uncertain asset prices, a decline in the overnight interest rate all the way to the reserve rate as well as an increase in bank leverage and an effective shutdown of interbank Federal Funds lending. However, we did not see a large deflation – after an initial small drop in late 2008 the price level remained quite stable over time.

An expansion of reserves

According to our model, a candidate for the absence of deflation is monetary policy. Suppose the Treasury issues a lot of new debt that is then purchased by the central bank in exchange for reserves. Suppose further that this is perceived as a one time change, with a stable path of nominal liabilities thereafter. In terms of the model, this policy corresponds to an increase in the outstanding nominal quantity of reserves.

In the abundant reserve regime, where reserves and other debt are perfect substitutes, an increase in reserves is neutral. The real interest rate and leverage do not change and the economy remains in the abundant reserves regime. The only variable that changes is the price level which rises in proportion to the increase in reserves. We conclude that an increase in uncertainty coupled with a large injection of reserves can move the economy into a period of abundant reserves with low asset prices, high leverage and low real rates, and the move is not accompanied by drop in the price level.

Unconventional monetary policy

An alternative response by central banks to a decline in asset values – a collateral shortage – has been to purchase low quality collateral, for example risky mortgage backed securities. We now consider what happens when the government purchases risky trees instead. We set up an experiment analogously to the open market purchase above. We start from an initial equilibrium with abundant reserves with reserves \( \tilde{M}_0 \), price level \( \tilde{P}_0 \) and leverage \( \ell \).

We assume that the government injects reserves to purchase all trees from banks’ balance sheet, that is, new reserves are chosen such that, at the new equilibrium with reserves \( M_0 \), price
Figure 8: Central bank purchase of trees when reserves are abundant

The effect of the purchase are just like in Figure 8. As trees are removed from bank balance sheets, the capital structure curve moves to the right.

After the additional injection, reserves continue to be abundant, so the policy has no effect on leverage (so \( \bar{\ell} = \ell \)) and real asset values. However, the policy does help stabilize the price level – it counteracts the deflationary effect of the increase in uncertainty. Indeed, collateral in the new equilibrium is \( M_0/\bar{P}_0 \) which equals the real value of old reserves \( M_0/P_0 \) plus the full real value of trees. In contrast, collateral in the initial equilibrium was given by the real value of reserves \( M_0/P_0 \) as well as the value of trees multiplied by the collateral quality weight \( \rho (s) < 1 \).

Unconventional policy thus works by replacing low quality real collateral on bank balance sheets with high quality nominal collateral. Since reserves continue to be abundant and the real return on reserves has not change, this does not actually lead to an increase in real collateral and a decline in leverage. However, backed by the new reserves, banks provide more nominal payment instruments which pushes up the price level. Compared with an outright increase in reserves, the inflationary effect of tree purchases is smaller since at the same trees are removed from the collateral pool.
Tree purchases by the central bank have two additional, more subtle, effects. First, like other increases in banks’ portfolio share of nominal assets discussed above, it makes the capital structure curve steeper. The slope in turn matters for the inflation response to changes in the interest rate on reserves: indeed, the steeper the capital structure curve, the less does an increase in the return on reserves push the price level down.

Second, removing trees from bank balance sheets reduces banks’ exposure to further shocks to asset quality. In particular, suppose that after all trees have been bought by the government, the uncertainty shock is reversed and asset prices increase. The payments system would not react to this shock as trees no longer serve as collateral to produce payment instruments. The economy would remain in an abundant reserves environment even though the asset market turbulence that has sent it there in the first place has actually subsided.

4.3 Carry traders

So far, the effect of asset values on bank balance sheet is direct: it requires bank investment in trees. In this section, we introduce institutional investors who borrow short term from banks in order to invest in trees. We call these investors carry traders – they do not actively trade trees but roll over their debt. This creates an additional link between securities markets and banks that operates even if banks only engage in short term lending. Moreover, monetary policy can affect carry traders’ funding cost.

Carry traders are competitive firms that issue equity, borrow overnight and invest in the subset of Θ∗, which is distinct from the subset accessible to banks. Like banks, carry traders face leverage costs, captured by an increase concave function c∗ that could be different from the cost function c_b assumed for banks.9 The leverage ratio of carry trader i is defined as overnight credit $F_{i,t}$ divided by the market value of the tree portfolio

$$\ell_{i,t} = \frac{F_{i,t}}{\int_{\Theta^*} Q_{i,t}^d dj}.$$  

Carry traders’ marginal collateral benefit and marginal cost of leverage are denoted $\kappa^*$ and $\lambda^*$, respectively.

We assume further that carry traders are more optimistic about the payoff of trees in $\Theta^*$ than households: they perceive uncertainty $s$ whereas households perceive $s^* > s$. The idea here is that the firm employs specialized employees who households trust to make asset management decisions. As a result, the spread relevant for investment in carry trader trees indirectly through investment by carry traders carries the uncertainty premium $s$ that makes these trees as desirable as other trees held directly by households.

*Optimal investment and borrowing*

Carry traders’ first order condition for overnight credit resembles that of banks in (A.6), except that overnight borrowing does not provide liquidity benefits: the return on equity must

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9We do not consider welfare effects of leverage for carry traders – instead we focus on the positive implications of margin trading. We thus assume for simplicity that leverage costs of carry traders are paid lump sum to households so that they have no impact on welfare.
be smaller than the real overnight rate plus the marginal cost of leverage. We focus on steady states only and drop time subscripts. Since we already know that the real rate is lower than \( \delta \) in equilibrium, it is always optimal for carry traders to borrow and we directly write the condition as an equality:

\[
\delta = i - \pi + \lambda^* (\ell^*_t) .
\]  

(18)

It follows that all carry traders choose the same leverage ratio and we drop subscripts from now on. Moreover, carry trader leverage is higher in equilibrium when interest rates are low.

Like banks, carry traders hold all trees accessible to them. This is due not only to the collateral benefit conveyed by trees, but also to carry traders’ relative optimism. The first order condition for tree \( j \) is

\[
\delta = r^j - s + \kappa^* (\ell^*_t) .
\]

When interest rates or uncertainty is low, carry traders apply a lower effective discount rate to trees, which results in higher tree prices.

The amount of carry trader borrowing in steady state equilibrium is

\[
F^* = \ell^* \frac{x^*}{\delta + s - \kappa^* (\ell^*)} .
\]

(19)

Here \( x^* \) represents total dividends on carry trader trees. Lower interest rates increase both leverage and the value of collateral and therefore increase borrowing. Moreover, an increase in uncertainty (that is, an increase in \( s \)) lowers collateral values and borrowing.

**Equilibrium with carry traders**

Our graphical analysis of equilibrium remains qualitatively similar when carry traders are added to the model. The only change is that carry trader borrowing now enters on the asset side of the banking sector. We can thus add in the denominator of the leverage ratio (13) a term \( B^* (\ell) \) that expresses carry trader borrowing as a function of bank leverage. We obtain the function \( B^* \) by substituting for \( \ell^* \) in (19) from (18) and then substituting for the interest rate from the bank first order condition from (A.10). The function \( B^* \) is increasing: if banks are more levered, the interest is lower and carry traders borrow more.

We can now revisit the effect of changes in beliefs and monetary policy in an economy with carry traders. Suppose first that, starting from an equilibrium with scarce reserves, there is an increase in uncertainty. The new effects is that, as carry traders value trees less, they demand fewer loans from banks. This increases bank leverage and shifts the capital structure curve to the right. The liquidity management curve does not change. In the new equilibrium, bank leverage is even higher and the interest rate is lower, as is the price level. The effect on prices therefore amplifies the increase in uncertainty about bank trees considered earlier. The additional prediction is that we should see a decline in funding of institutional investors via short term credit from payments intermediaries, such as a decline in repo extended by money market mutual funds to broker-dealers.

It is also interesting to reconsider the effect of monetary policy. Suppose policy engineers a change in the mix of bank assets or their value that lowers the real overnight interest rate. Carry trader borrowing increases and carry traders bid up the prices of the trees they invest in. As one segment of the tree market thus increases in value, the aggregate value of trees also
rises: there is a tree market boom. Importantly, this is not a standard real interest rate effect: the discount rate of households, which is used to value trees held by households, is unchanged. The effect comes solely from the effect of monetary policy on the overnight rate and hence on carry traders’ funding costs.

4.4 Active traders

Carry traders provide collateral to banks and thus affect the supply of payment instruments. We now introduce another group of institutional investors who demand payment instruments. Active traders periodically rebalance their portfolios as their view of tree payoffs changes, and they require cash to pay for new trees. The demand of active traders for payment instruments counteracts the supply side effects due to changes in the collateral values or the borrowing by carry traders. At the same time, monetary policy also affects their funding costs.

Active traders are competitive firms that issue equity and invest in payment instruments as well as a subset of trees $\Theta$. There are many active traders and each is optimistic about one particular “favorite” tree: the trader perceives uncertainty $s$ about this tree. In contrast, households and other active traders perceive uncertainty $\hat{s} > s$. Active traders also perceive $\hat{s} > s$ about all other trees. Every period, the identity of the favorite tree within the subset $\hat{\Theta}$ changes with probability $\hat{v} \leq 1$ to some other tree in the subset.

To generate a need for payment instruments, we assume that active traders who change their tree position must pay for the new tree purchases with prearranged payment instruments or intraday credit. Active trader $i$ faces the liquidity constraint

$$z_{i,t} \sum_{\Theta} Q^i_t \hat{\theta}_{i,j,t} = I_{i,t} + (\hat{D}_{i,t} + \hat{L}_{i,t}),$$

where $z_i$ is a random variable that indicates whether or not the favorite tree of active trader $i$ has changed in the current period, $I_i$ is the intraday credit position, $\hat{D}_i$ are deposits that the fund keeps at its bank together with credit lines $\hat{L}_i$.

Like banks, active traders $i$ faces a limit on intraday credit

$$I_{i,t} \leq \hat{\gamma}(\hat{D}_{i,t} + \hat{L}_{i,t}),$$

where $\hat{\gamma}$ is a parameter that governs netting in tree transactions. It is generally different from the parameter $\gamma$ that governs netting among banks, since it captures netting by a clearing and settlement system for the securities that active traders invest in.

Active traders choose payment instruments, trees and their shareholder payout. We focus on equilibria in which every active trader always holds only its favorite tree – we can assume that the perceived uncertainty on other trees is high enough. Since payment instruments are costly – the real rate on deposits is below the discount rate – active traders arrange as few payment instruments as necessary in order to purchase the entire outstanding amount of their new favorite tree in case the identity of their favorite tree changes. It follows that the intraday credit limit binds in equilibrium, a form of "cash-in-the-market pricing".

Optimal investment and deposits
To make the marginal benefit of payment instruments equal to the discount rate, payment instruments must provide a liquidity benefit. The latter comes from traders’ ability to invest in their favorite tree, which carries a return that compensates them for the opportunity cost of deposits. Indeed, in steady state, the first order conditions for deposits and trees simplify to

\[ \delta = \hat{i}_D - \pi + (1 + \hat{\gamma})(r_j - s - \delta). \]

The equilibrium return \( r_j \) adjusts to provide an excess return \( r_j - s - \delta \) on the favorite trees. Since every trader perceives a lower spread on his own favorite tree compared to any other tree, this excess return can persist in equilibrium.

Equilibrium payment instruments are proportional to the market value of active traders’ favorite trees:

\[ (\hat{D} + \hat{L})(1 + \hat{\gamma}) = \frac{\hat{x}P}{\delta + s + \frac{\delta - (i_D - \pi)}{1 + \hat{\gamma}}}, \]

where \( \hat{x} \) represents dividends on active trader trees. The market value of trees, as well as deposit holdings, respond to the opportunity cost of trading: if the deposit rate is higher, then active traders earn lower returns on their trees, prices are higher and active traders hold more deposits. Moreover, an increase in uncertainty lowers asset values and the demand for deposits.

**Equilibrium with active traders**

We focus on local changes to equilibria with abundant reserves. Our graphical analysis of equilibrium then remains qualitatively similar when active traders are added to the model. What changes is that transactions now include carry traders’ asset purchases:

\[ T = \Omega + x + \hat{\nu}(\hat{D} + \hat{L})/P. \]

Since we know that the equilibrium deposit rate is decreasing in banks’ leverage and liquidity ratio, we can replace the exogenous ratio \( T \) in the leverage ratio (13) by a function \( T(\ell) \) that is decreasing in leverage.

We can now revisit the effect of changes in beliefs and monetary policy in an economy with active traders. Suppose first that there is an increase in uncertainty. As active traders value trees less, they demand fewer payment instruments. This lowers bank leverage and shifts the capital structure curve to the left. The liquidity management curve does not change. In the new equilibrium, bank leverage is lower and the interest rate is higher, as is the price level. In other words, active traders are a force that generate the opposite response to a change in uncertainty from banks and carry traders. Since in the typical economy all traders are present to some extent, we can conclude that their relative strength is important. An additional prediction is that we should see a decline in payment instruments – either deposits or credit lines – provided to institutional investors.

We can also reconsider the effect of monetary policy. Suppose once more that policy lowers the real overnight interest rate. The opportunity cost of holding deposits falls and active traders demand more payment instruments. At the same time, they bid up the prices of the trees they invest in. Again a segment of the tree market increases in value and the aggregate value of trees also rises: there is a tree market boom. Again the effect is not due a change in the discount rate, but instead a change in the funding cost: here it affects active traders’ strategy which requires payment instruments in order to wait for trading opportunities.


A Appendix

In this appendix we derive the main equation characterizing equilibrium. Section A.1 derives banks’ first order conditions. Section A.2 derives a system of equations characterizing equilibrium.

A.1 Bank optimization

This section studies the optimal choice of banks that maximize (3) subject to (7), 4) and (9).

Paying for leverage costs, marginal leverage costs and marginal collateral benefit

Consider first banks’ choice of credit line to pay for leverage costs. Credit lines granted by other banks do not contribute to collateral and hence do not show up in the leverage ratio (8). As long as the interest rate on credit lines is positive, the constraint then binds in equilibrium: banks arrange for a line that is just large enough to cover the leverage costs that will accrue next period.

Using the budget constraint and the binding liquidity constraint, the last two terms in the bank budget equation (7) describe the cost of leverage chosen in the previous period and can be written as

\[ -e^{\pi t} (1 + \frac{L_{t-1}}{v}) C_b(\ell_{t-1}) (\sigma(D_t + L_t) + F_t). \]  

Since banks have to arrange a credit line, their effective cost of leverage thus also includes the cost of the line.

To derive bank first-order conditions, it is helpful to define the marginal cost of leverage as the derivative of the discounted effective leverage cost (A.1) with respect to total commitments (the numerator in (8)) and the marginal benefit of collateral as the (discounted) derivative with respect to the denominator:

\[
\lambda_t = e^{-\hat{\delta} t} \left( c_b(\ell_t) + c'_b(\ell_t) \ell_t \right) \left\{ 1 + \frac{i_t^L}{v} \right\}, \\
\kappa_t = e^{-\hat{\delta} t} c'_b(\ell_t) \ell_t^2 \left\{ 1 + \frac{i_t^L}{v} \right\}. 
\]  

An extra unit invested in assets at date \( t \) that contributes to collateral earns not only a pecuniary return, but also the collateral benefit \( \kappa_t \). Similarly, committing to a unit payable entails the extra leverage cost \( \lambda_t \). Concavity implies that both \( \lambda_t \) and \( \kappa_t \) are increasing in leverage since \( 2c'_b(\ell) + c''_b(\ell) \ell > 0 \). Since the cost function \( c \) slopes up sufficiently fast that banks always choose \( \ell < 1 \), we also have that for a given level of leverage, the marginal cost of leverage is higher than the marginal collateral benefit: \( \lambda_t > \kappa_t \).

Bank first-order conditions for assets

The typical bank’s first-order conditions describe the key trade-offs of portfolio and capital structure choice. Since shareholders are risk neutral, the expected marginal benefits or costs of all assets and liabilities are compared to the required return on equity \( \hat{\delta} \). If the portfolio of the bank is chosen optimally, the marginal benefit from any asset cannot be larger than \( \hat{\delta} \): if not, then the bank would choose to invest more. Moreover, the marginal benefit is equal to \( \hat{\delta} \) if the
bank optimally holds a positive position in the asset. If the marginal benefit is below \( \hat{\delta} \) then the bank does not invest: it is better to pay dividends instead.

Consider the first-order condition for overnight lending which not only earns the real interest rate, but also the collateral benefit. The return on equity must be higher than the marginal benefit:

\[
\hat{\delta}_t \geq i_t - \pi_{t+1} + \kappa_t,
\]  

with equality if the bank lends overnight. In the latter case, low real interest rates imply that overnight credit is valuable collateral, so banks optimally choose higher leverage. Put differently, highly levered banks obtain a high benefit from overnight lending as collateral and thus require a lower return on credit.

The first-order condition for trees is similar. The real rate of return on tree \( j \) held by banks is \( r^j_{t+1} = \log(Q^j_{t+1} + P_{t+1}x_{t+1})/Q^j_t - \pi_{t+1} \). Since trees also deliver collateral benefits, we must have

\[
\hat{\delta}_t \geq r^j_{t+1} + \rho \kappa_t,
\]

with equality for trees held by the bank. Since the collateral benefit lowers the return on trees held by the bank, it raises the price of the trees relative to payoff. Indeed, holding fixed the payoff \( Q^j_{t+1} + P_{t+1}x_{t+1} \), the price of a tree held by the bank is \( Q^j_t = (Q^j_{t+1} + P_{t+1}x_{t+1})/\left(\hat{\delta}_t - \rho \kappa_t\right) \).

In particular, if banks holding tree \( j \) are more levered, then those trees are more valuable collateral, their cash flows are discounted at a lower rate and their price is higher.

Reserves differ from overnight lending and trees in that they not only provide returns \( i^R_t - \pi_{t+1} \) and collateral benefits, but also liquidity benefits – they can be used for payments. The liquidity benefit depends on the Lagrange multiplier on the liquidity constraint (4). Writing \( \mu_t \) for that Lagrange multiplier divided by the price level, the first order condition for reserves is

\[
\hat{\delta}_t \geq i^R_t - \pi_{t+1} + \kappa_t + (1 + \gamma) e^{-\hat{\delta}_t - \pi_{t+1}} E \left[ \mu_{t+1} \right].
\]  

(A.4)

The liquidity benefit (the second term) is higher the higher the expected discounted Lagrange multiplier \( E \left[ \mu_{t+1} \right] \) and the more payments can be made per dollar of reserves (higher \( \gamma \)).

**Bank first-order conditions for liabilities & credit lines**

Consider now banks’ choice to finance themselves with deposits or overnight credit. If the capital structure of the bank is chosen optimally, the marginal cost of any liability type cannot be lower than \( \hat{\delta}_t \), the cost of issuing equity: if not, banks would choose to borrow more. Moreover, the marginal cost is equal to \( \hat{\delta}_t \) if the bank holds a positive position in that particular liability type. If the marginal cost is above \( \hat{\delta}_t \) then the bank does not issue the liability: it is better to issue equity instead.

The first-order condition for deposits says that the equity return must be smaller than the sum of the real deposit rate plus the marginal leverage and liquidity costs of deposits

\[
\hat{\delta}_t \leq i^D_t - \pi_{t+1} + \sigma \lambda_t + e^{-\hat{\delta}_t - \pi_{t+1}} (1 + \gamma) E \left[ \mu_{t+1} \phi_{t+1} \right],
\]  

(A.5)

with equality if the bank issues deposits. Leverage costs increase with overall leverage through \( \lambda_t \) and are also scaled by the parameter \( \sigma \) which makes deposits cheaper than other borrowing.
Liquidity costs arise in those states next period when positive liquidity shocks $\phi_{t+1} > 0$ coincide with a binding intraday credit limit $\mu_{t+1} > 0$.

The first-order condition for overnight borrowing says that the equity return must be smaller than the sum of the real overnight rate plus the marginal leverage cost, less the liquidity benefit provided by overnight credit:

$$\delta_t \leq i_t - \pi_{t+1} + \lambda_t - \mu_t (1 + \gamma).$$

(A.6)

For banks that borrow overnight, the condition holds with equality. This can happen because for those banks the intraday credit limit binds and $\mu_t > 0$. In contrast, banks with sufficient reserves have $\mu_t = 0$ and do not borrow.

Finally, consider banks’ choice to extend credit lines, a form of implicit liability. The fee earned for extending the line must at least compensate the bank for the leverage cost occurred as well as the expected liquidity costs

$$i_t^L \geq \sigma \lambda_t + \delta_e^{-\delta_t - \pi_{t+1}} E \left[ \mu_{t+1} \phi_{t+1} \right].$$

(A.7)

As discussed above, we focus on equilibria with $i_t^D - \pi_{t+1} = \delta_t - i_t^L$ so that deposits and credit lines are equivalent from the perspective of households. Comparing (A.5) and (A.7), the two payment instruments are then also equivalent from the perspective of banks.

### A.2 Characterizing equilibrium

In this section, we derive a difference that summarizes the equilibrium dynamics. The variables are the interest rates $i_t$ and $i_t^L$, tree prices $Q_t^i$, the nominal price level $P_t$, aggregate payment instruments $D_t + L_t$, the liquidity ratio $\phi_t^*$ (which is inversely proportional to the "money multiplier" $(D_t + L_t)/M_t$), the leverage ratio $\ell_t$, the marginal cost of leverage $\lambda_t$, the marginal benefit of collateral $\kappa_t$ and the marginal benefit of liquidity $\tilde{\mu}_t$ which is also equated across banks in equilibrium.

We use the notation $\tilde{\mu}$ to denote the liquidity benefit that obtains per real dollar for any bank for which the liquidity constraint binds. The notation $\mu$ is the Lagrange multiplier n the first order condition of an individual bank (A.4). Of course, in the cross section of banks, there are banks for whom the constraint does not bind and for those banks we have $\mu = \tilde{\mu} = 0$.

#### Asset pricing

Consider participation in overnight credit and tree markets. In an equilibrium with positive consumption, banks must supply payment instruments. Since payment instruments entail leverage costs, banks obtain a positive collateral benefit from assets they are eligible to hold. As a result, households do not invest in any asset markets that banks can invest in: at least one bank will bid up the price of any asset accessible to banks until its return is below the discount rate and the asset is unattractive to households.

Leverage is the same for all active banks in equilibrium. The equilibrium rate of return on
a tree $j$ accessible to banks is then related to aggregate leverage by the bank “Euler equation”

$$\delta_t = r^j_{t+1} + \rho \kappa_t.$$  \hspace{1cm} (A.8)

The flip side of lower returns is higher prices. Indeed, let $x^B_t$ denote the sum of dividends on all trees accessible to banks at date $t$ and let $v^B_t = \int_{Q^B_t} Q^j_t dj / P_t$ denote the real value of those trees. In equilibrium, the value of these bank trees reflects not only the present value the future payoff on the tree, but also the collateral value:

$$v^B_t = e^{-\delta_t} (v^B_{t+1} + x^B_{t+1} + \rho \kappa_t v^B_t).$$  \hspace{1cm} (A.9)

Using our conventions, pricing effectively works as if payoffs are discounted at the lower rate $\hat{\delta}_t - \rho \kappa_t$.

**Liquidity benefits and reserve scarcity**

Bank liquidity management and their activity in the overnight credit market depends crucially on the scarcity of reserves. Indeed, with abundant reserves, the first order condition for reserves (A.4) implies that the real rate on reserves is equal to $\hat{\delta}_t - \kappa_t$. From the first order condition for overnight lending (A.3), this is the same overnight interest rate that obtains if banks lend in the overnight market – if reserves are abundant, they are a perfect substitute to overnight paper and must earn the same interest rate.$^{11}$

The overnight interest rate is connected to leverage through a bank Euler equation analogous to (A.8),

$$\hat{\delta}_t - (\lambda_t - \pi_{t+1}) = \kappa_t.$$  \hspace{1cm} (A.10)

As long as there is some government debt $B_t$, this equation holds in any equilibrium, whether reserves are abundant or not. At the same time, the inequality (A.3) implies that households never participate in the overnight market – as with banks trees, banks bid up the price of overnight paper to the point where the asset is unattractive to households. In particular, a regime of abundant reserves has the property that reserves and overnight credit are perfect substitutes from the perspective of banks, but neither asset is ever held by households.

Consider a bank that must borrow at the current date, that is, it receives a liquidity shock $\phi_t$ beyond the liquidity ratio $\phi_t^* = M_t (1 + \gamma) / \bar{v}(D_t + L_t)$ from (5). The bank’s liquidity benefit from overnight credit follows from (A.3) and (A.6):

$$\bar{\mu}_t (1 + \gamma) = \lambda_t - \kappa_t > 0.$$  \hspace{1cm} (A.11)

Tapping the overnight market entails both a leverage cost, and an opportunity cost in terms of foregone collateral value. A more leveraged banking system thus faces a larger penalty of running out of reserves.

$^{11}$Of course, reserves still flow across banks. In particular an amount of reserves $\bar{\phi} \bar{v} (D + L) / (1 + \gamma)$ still serves to buffer liquidity shocks. However reserves beyond this amount are equivalent to overnight paper that cannot be used to handle payments instructions.
The spread between overnight and reserve rate reflects the expected liquidity benefit of holding reserves. Substituting into the first-order condition for reserves, we obtain

\[ i_t - i_t^R = (1 - G(\phi_{t+1}^*)) e^{-\delta_t - \eta_{t+1}} \mu_{t+1}. \] (A.12)

A liquidity benefit obtains only when the withdrawal shock exceeds \( \phi_{t+1}^* \), that is, with probability \( 1 - G(\phi_{t+1}^*) \). In this case, a bank holding an extra dollar of reserves saves the excess cost of overnight lending relative to equity.

In equilibria with scarce reserves \( i_t > i_t^R \), the spread (A.12) shows how banks choose the reserve-deposit ratio as a function of interest rates and leverage. The interest rate \( i \) plays a dual role here. On the one hand, it affects the opportunity cost of reserves. Indeed, holding fixed the liquidity benefit, the equation works much like a money demand equation: if the overnight rate is higher, then it is more costly to hold reserves and banks choose smaller \( \phi_t^* \), or fewer reserves per dollar of deposits and credit lines. On the other hand, the interest rate affects the liquidity benefit itself: holding fixed leverage \( \ell_t \) and the spread, a higher interest rate increases the liquidity benefit of reserves and leads banks to choose more reserves to avoid the higher penalty of running out.

**Liquidity and leverage**

Bank leverage depends on how much of each source of collateral is available, and how much banks must borrow apart from issuing deposits. In particular, interbank credit contributes both to collateral (for lender banks) and raises costs (for borrower banks). Given leverage \( \ell_t \) and a liquidity ratio \( \phi^* \leq \tilde{\phi} \), the equation for interbank borrowing (6) delivers the ratio of outstanding interbank credit to transactions

\[
\frac{F_t}{D_t + L_t} = \frac{\tilde{v}}{1 + \gamma} \int_{\phi_t^*}^{\phi_t} (\phi - \phi_t^*) dG(\phi) =: \frac{\tilde{v}}{1 + \gamma} f(\phi_t^*). 
\] (A.13)

The function \( f \) is decreasing in \( \phi^* \): if interest rates are such that banks hold a lot of reserves, then \( \phi^* \) is high and banks rarely run out of reserves, so outstanding interbank credit is low. In this sense, reserves and overnight are substitutes in liquidity management.

Collecting promises due to payment instruments, real reserves, trees and interbank credit, as well as collateral in the form of reserves, government debt, trees, and interbank credit, equilibrium leverage satisfies

\[
\ell_t = \frac{\sigma (D_{t+1} + L_{t+1}) + (D_t + L_t) \frac{\phi_t^*}{1 + \gamma} f(\phi_t^*)}{M_{t+1} + B_{t+1}^G + P_t \rho_t^{t'} + (D_t + L_t) \frac{\phi_t^*}{1 + \gamma} f(\phi_t^*)}. 
\] (A.14)

Holding fixed the value of collateral from outside the banking system – reserves, government debt and trees – scarcity of reserves requires more leverage to support a given quantity of transactions. Indeed, we have assumed a cost function \( c_b \) such that the economy operates in the range \( \ell \leq 1 \); the presence of an interbank market adds an equal amount of debt and collateral and hence increases leverage.

**The cost of payment instruments**

From banks’ first-order condition, the equilibrium rate on credit lines satisfies

\[
i_t^b = \sigma \lambda_t + \frac{\tilde{v}}{1 + \gamma} e^{-\delta_t - \eta_{t+1}} \bar{\mu}_{t+1} \int_{\phi_{t+1}^*}^{\phi_t} \phi dG(\phi) .
\] (A.15)
The two terms represent a leverage and a liquidity component. When reserves are abundant, we have \( \phi_{t+1}^* > \bar{\phi} \) so the liquidity component is zero and the cost of a credit line simply reflects banks’ cost of leverage.

When reserves are scarce, banks incur additional costs when they run out of reserves. Those costs are larger if velocity is higher and there is less netting among banks. They further depend on the marginal benefit of liquidity as well as on the liquidity shock the bank receives in the next period. The costs banks incur when providing payment instruments also lower the deposit rate \( i_t^D - \pi_{t+1} \) by the same amount.\(^{12}\)

**Equilibrium**

An equilibrium is characterized by the seven equations (10) and (A.9)-(A.12) and (A.14)-(A.15) together with equation (5) that defines the liquidity ratio as well as the two equations (A.2) that define marginal leverage cost and collateral benefit. If policy is described by a path for the money supply, then no additional equations are needed. The ten equations determine ten endogenous variables \( i, i_L, v_B, P, D + L, \ell, \phi^*, \lambda, \kappa \) and \( \bar{\mu} \). The difference equation characterizing equilibrium has only two state variables: aggregate nominal reserves \( M_t \) and aggregate nominal payment instruments \( D_t + L_t \).

Alternatively, we can consider equilibria that obtain when policy is described by an interest rate rule. Since output is exogenous, we focus on rules that depend on inflation, \( i_t = g(\pi_t) \) for some function \( g \). The interest rate rule then provides an additional equation, and \( M_t \) is an additional endogenous variable. Since the interest rate rule conditions on inflation at the previous date, the past price \( P_{t-1} \) also becomes part of the initial conditions of the difference equation.

In order to compute equilibrium consumption and welfare at date 0, we need to know additional predetermined variables: initial leverage ratios \( \ell_{t-1} \) and \( \ell^p_{t-1} \) for banks and the government, respectively as well as banks’ outstanding overnight borrowing \( F_0 \) and lending \( B_0 \). Those variables are needed to compute the real resources \( c_G(\ell^p_{t-1}) M_0/P_{t-1} \) and \( c_B(\ell_{t-1}) (D_0 + L_0 + F_0)/P_{t-1} \) purchased by the government and banks to pay leverage costs at date 0. At the same time, they do not affect any endogenous variables other than consumption which does not enter the difference equation; as a result there is no need to treat them explicitly as state variables.

Consider equilibria in which reserves are abundant at all times, so \( \phi_{t+1}^* \geq \bar{\phi} \) for all \( t \). The three liquidity management equations (A.11)-(A.13) are then redundant, the variables \( \phi^* \) and \( \mu \) can be removed from the system and the overnight interest rate achieves its lower bound \( i = i_R \). The remaining six equations then determine \( i_L, v_B, P, \ell, \lambda \) and \( \kappa \).

**Approximating equilibria with small rates of return**

We have simplified formulas above by assuming that rates of return are small decimals so their products can be ignored. To guarantee that small rates to obtain in equilibrium, we scale the leverage cost function and bound the real rate on reserves. Let \( \bar{r}_R \) denote a lower bound on the real rate on reserves, a small decimal number. In what follows, we guarantee this bound by appropriate assumptions on monetary policy.

\(^{12}\) The term "deposit rate" should be interpreted broadly here: it is the only cost endusers pay for payments services in our model, since we do not explicitly model other costs such as account and transaction fees.
The highest possible leverage ratio that can obtain in any equilibrium then satisfies
\[
\delta - \bar{r}_R = c_b(\ell) \bar{\ell}^2 (1 + \sigma (c_b(\ell) + c_b'(\ell) \bar{\ell})).
\]
This leverage ratio corresponds to an equilibrium where the reserve rate hits the bound and reserves are abundant. If reserves were scarce, the benefit of reserves would include liquidity components and leverage would have to be lower.

We now choose the cost function \( c \) such that \( \bar{\ell} < 1 \) and \( c'(\bar{\ell}) \bar{\ell} \) is a small decimal number, much like \( \delta - \bar{r}_R \). It follows that the marginal cost of leverage \( \lambda \) and the marginal benefit of collateral \( \kappa \) are also small decimal numbers in any possible equilibrium. Moreover, the effect of the interest rate on credit lines on \( \lambda \) and \( \kappa \) is second order. We thus use the additional approximation
\[
\begin{align*}
\lambda(\ell) &= c_b(\ell) + c_b'(\ell) \ell, \\
\kappa(\ell) &= c_b'(\ell) \ell^2,
\end{align*}
\]
where both \( \lambda \) and \( \kappa \) are increasing function of leverage. We thus abstract from effects of the interest rate \( i_L \) on banks’ cost of leverage. Those effects are small and not economically interesting; omitting them altogether makes for cleaner formulas below.

The fact that equilibrium \( \lambda \) and \( \kappa \) are small has several convenient implications. First, we can omit the factor \( e^{\delta - \pi} \) on the right hand sides of (A.12) and (A.15). The timing of the liquidity benefit has only a second order effect. Second, combining (A.16) and (A.10), we obtain a one-to-one relationship between the real overnight interest rate and leverage: banks increase leverage if interest rates are lower. We use both properties when characterizing equilibria further below.

### A.3 Steady state

This section derives the equations characterizing steady state. We assume the same exogenous growth rate \( \pi \) for the nominal quantities \( M_t \) and \( B_t^\pi \) so the ratio \( B_t^\pi / M_t \) is constant over time. With constant rates of return, the marginal rate of substitution of wealth across dates equals the discount rate, that is \( \delta_t = \delta \). Moreover, the key ratios chosen by banks, leverage \( \ell \) and the liquidity ratio \( \phi^* \), are constant over time.

Since output is fixed, payment instruments and the price level also grow at the rate \( \pi \). Using the quantity equation (10), the constant liquidity ratio \( \phi^* = M_t/D_t \) can be written as
\[
\phi^* = \frac{1 + \gamma}{T} m.
\]

The steady state real amount of overnight credit is also linked directly to transactions and reserves. Combining (A.13) and (A.17), we have
\[
\frac{F_t}{P_t} = \frac{T}{1 + \gamma} \tilde{f} \left( \frac{1 + \gamma}{T} m \right) = f(m; \gamma),
\]

7
The Euler equation 12a for the overnight real interest rate follows directly from (A.10). The steady value of bank trees is found from (A.9) as

\[ v_B = \frac{x_B}{\delta - \kappa(\ell)}. \]

Consider now the capital structure curve relationship (13). Substituting (10), (A.9) and (11) into (A.14), we find that steady state real reserves and leverage are linked by

\[ \ell = \frac{\sigma T/\bar{\ell} + f(m; \gamma)}{m(1 + B^p_t/M_t) + \rho \delta - \kappa(\ell) + f(m; \gamma)}. \]

In a steady state with nonzero inflation \( \pi \), overnight credit \( f \) should be premultiplied by \( e^{\pi} \). We omit this factor here in line with our focus on small rates of return.

Consider now the equations describing liquidity management. To derive the liquidity management curve (14), we substitute (A.10) and (A.11) into (A.12) and obtain

\[ \delta - (i^R - \pi) = \kappa(\ell) + (1 - G(m(1 + \gamma)/T))(\lambda(\ell) - \kappa(\ell)). \]

Finally, to derive equation (15) – used in the text to discuss interest rate policy – we substitute (A.10) and (A.11) into (A.12) to get

\[ i - i^R = (1 - G(m(1 + \gamma)/T))(\lambda(\ell) - \kappa(\ell)). \]
References


