Speculation and Risk Sharing with New Financial Assets

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Abstract

I investigate the effect of financial innovation on portfolio risks when traders have belief disagreements. I decompose traders' average portfolio risks into two components: the uninsurable variance, defined as portfolio risks that would obtain without belief disagreements, and the speculative variance, defined as portfolio risks that result from speculation. My main result shows that financial innovation always increases the speculative variance through two distinct channels: By generating new disagreements, and by amplifying traders' speculation on existing disagreements. When disagreements are large, these effects are sufficiently strong that financial innovation *increases* average portfolio risks. Moreover, a profit seeking market maker *endogenously* introduces assets that *increase* average portfolio risks.

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1 Introduction

According to the traditional view of financial innovation, new financial assets facilitate the diversification and the sharing of risks.[1] However, this view does not take into account that new assets are often associated with much uncertainty, especially because they do not have a long track record. Belief disagreements come as a natural by-product of this uncertainty and change the implications of risk taking in these markets. In particular, market participants' disagreements about how to value new assets naturally lead to speculation, which represents a powerful economic force that tends to increase risks.

An example is offered by the recent crisis. Assets backed by pools of subprime mortgages (e.g., subprime CDOs) became highly popular in the run-up to the crisis. One role of these assets is to allocate the risks to market participants who are best able to bear them. The safer tranches are held by investors that are looking for safety (or liquidity), while the riskier tranches are held by financial institutions who are willing to hold these risks at some price. While these assets (and their CDSs) should have served a stabilizing role in theory, they became a major trigger of the crisis in practice, when a fraction of financial institutions realized losses from their positions. Importantly, the same set of assets also generated considerable profits for some market participants,[2] which suggests that at least some of the trades on these assets were speculative. What becomes of the risk sharing role of new assets when market participants use them to speculate on their different views?

To address this question, this paper analyzes the effect of financial innovation on portfolio risks in a model that features both the risk sharing and the speculation motives for trade. Traders with income risks take positions in a set of financial assets, which enables them to share and diversify some of their background risks. However, traders have belief disagreements about asset payoffs, which induces them to take also speculative positions on assets. I assume traders have mean-variance preferences over net worth. In this setting, a natural measure of portfolio risk for a trader is the variance of her net worth. I define *the average variance* as an average of this risk measure across all traders. I further decompose the average variance into two components: *the uninsurable variance*, defined as the variance that would obtain if there were no belief disagreements, and *the speculative variance*, defined as the residual amount of variance that results from speculative trades based on belief disagreements. I model financial innovation as an expansion of the set of assets available for trade. My main result characterizes the effect of financial innovation on each component of the average variance. In line with the traditional view, financial innovation always decreases the uninsurable variance because new assets increase the possibilities for risk sharing. Theorem 1 shows that financial innovation also always increases the speculative variance. Moreover, when belief disagreements are sufficiently

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[1] Cochrane (2001) summarizes this view as follows: "Better risk sharing is much of the force behind financial innovation. Many successful new securities can be understood as devices to more widely share risks."

[2] Lewis (2010) provides a detailed description of investors that took a short position on housing related assets in the run-up to the recent crisis.

large, this effect is sufficiently strong that financial innovation increases the average variance.

My analysis identifies two distinct channels by which financial innovation increases the speculative variance. First, new assets lead to new disagreements because they are associated with new uncertainties. Second, and more subtly, new assets also amplify speculation on existing disagreements. To illustrate the second channel, Theorem 1 shows that new assets increase the speculative variance even if traders completely agree about their payoffs. The intuition behind the second channel is a powerful economic force: *the hedge-more/bet-more* effect. To see this effect, consider the following example. Suppose two traders have different views about the Swiss Franc, which is highly correlated with the Euro. The optimist believes the Franc will appreciate while the pessimist believes it will depreciate. Traders do not disagree about the Euro, perhaps because they disagree about the prospects of the Swiss economy but not the Euro zone. First suppose traders can only take positions on the Franc. In this case, traders may not take too large speculative positions because the Franc is affected by several sources of risks some of which they don't disagree about. Traders must bear all of these risks which might make them reluctant to speculate. Suppose instead the Euro is also introduced for trade. In this case, traders complement their positions in the Franc by taking the opposite positions in the Euro. By doing so, traders hedge the risks that also affect the Euro (on which they agree), which enables them to take purer bets on the Franc. When traders are able to take purer bets, they also take larger bets. Theorem 1 shows that this hedge-more/bet-more effect is sufficiently strong that the introduction of the Euro in this example (and more generally, any new asset) increases the speculative variance.

Theorem 1 takes the new assets as exogenous and analyzes their impact on portfolio risks. In practice, new financial assets are endogenously introduced by economic agents with profit incentives. A sizeable literature emphasizes risk sharing as a major driving force in endogenous financial innovation [see, for example, Allen and Gale (1994) or Duffie and Rahi (1995)]. A natural question is to what extent the risk sharing motive for financial innovation is robust to the presence of belief disagreements. I address this question by introducing a profit seeking market maker that innovates new assets for which it subsequently serves as the intermediary. The market maker's expected profits are proportional to traders' willingness to pay to trade the new assets. Thus, traders' speculative trading motive, as well as their risk sharing motive, creates innovation incentives for the market maker. In particular, the optimal asset design depends on the size and the nature of belief disagreements, in addition to the risk sharing possibilities. When traders have common beliefs, the market maker innovates assets that minimize the average variance, as in Demange and Laroque (1995) and Athanasoulis and Shiller (2000, 2001). In contrast to these traditional results, Theorem 2 also characterizes the polar opposite case: When traders' belief disagreements are sufficiently large, the endogenous new assets *maximize* the average variance among all possible choices. Intuitively, the market maker innovates assets that enable traders to bet most precisely on their belief disagreements, completely disregarding the risk sharing motive for financial innovation. Taken together, Theorems 1 and

2 establish that belief disagreements can substantially change the effect of financial innovation on portfolio risks, as well as the driving force behind some of those innovations.

A natural question concerns the normative implications of Theorems 1 and 2. In my baseline setting, financial innovation leads to a Pareto improvement despite the fact that it might increase portfolio risks. This is because each trader perceives a large expected return from her speculative positions in new assets, which justifies the additional risks that she is taking. There are four caveats to note about this observation. First, the reason for the efficiency of financial innovation in this setting is almost the opposite of what has been emphasized in much of the previous literature. In particular, when belief disagreements are sufficiently large, traders' welfare gains do not come from a decrease in their risks, but from an increase in their perceived expected returns. Second, and relatedly, it is not clear whether these perceived expected returns should be viewed as welfare gains, because they are driven by belief disagreements. In particular, while all traders perceive high expected returns, at most one of these expectations can be correct. In fact, a large and growing theory literature has argued that Pareto efficiency is not the appropriate welfare criterion for environments with belief disagreements (see Brunnermeier, Simsek, and Xiong, 2012 and the references therein). In Section 6.1, I show that the equilibrium is not necessarily efficient according to an alternative welfare criterion, which is particularly appropriate when belief disagreements are viewed as distortions emerging from psychological biases. Third, if the environment is extended to feature *externalities*, then financial innovation might be inefficient even according to the Pareto criterion. In Section 6.2, I show that externalities and associated inefficiencies naturally emerge when traders are viewed as financial intermediaries under (explicit or implicit) government protection. Importantly, in both Sections 6.1 and 6.2, a measure of traders' average portfolio risks, similar to that characterized in Theorems 1 and 2, emerge as the central object in welfare analysis. Fourth, these analyses should be viewed as partial exercises, characterizing the welfare effects of financial innovation that operate through portfolio risks. In particular, I do not take a strong normative stand in this paper since financial innovation might also impact welfare through various other channels missing from my analysis. Among other things, even pure speculation driven by financial innovation can provide some social benefits by making asset prices more informative.[3]

As the above discussion suggests, my paper belongs to a sizeable literature on financial innovation and security design [see, in addition to the above-mentioned papers, Van Horne (1985), Miller (1986), Ross (1988), Merton (1989), Gorton and Pennacchi (1990), Cuny (1993), Duffie and Jackson (1989), Demarzo and Duffie (1999), Tufano (2003)]. This literature, with

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[3]On the other hand, there is also a large literature that emphasizes various other channels by which financial innovation can reduce welfare. Hart (1975) and Elul (1994) show that new assets that only partially complete the market may make all agents worse off in view of general equilibrium price effects. Stein (1987) shows that speculation driven by financial innovation can reduce welfare through informational externalities. I abstract away from these channels by focusing on an economy with single good (hence, no relative price effects) and heterogeneous prior beliefs (hence, no information). More recently, Rajan (2005), Calomiris (2008), and Korinek (2012) argue that financial innovation might exacerbate agency problems, Gennaioli, Shleifer and Vishny (2010) emphasize neglected risks associated with new assets, and Thakor (2010) emphasizes the unfamiliarity of new assets.

the exception of a few recent papers (some of which are discussed below), has not explored the implications of belief disagreements for financial innovation. For example, in their survey of the literature, Duffie and Rahi (1995) note that "one theme of the literature, going back at least to Working (1953) and evident in the Milgrom and Stokey (1982) no-trade theorem, is that an exchange would rarely find it attractive to introduce a security whose sole justification is the opportunity for speculation." The results of this paper show that this observation does not apply if traders' belief differences reflect their *disagreements* as opposed to private information.

Within the financial innovation literature, my paper is most closely related to the work of Brock, Hommes, and Wagener (2009), who also emphasize the hedge-more/bet-more effect and identify destabilizing aspects of financial innovation. The papers are complementary in the sense that they use different ingredients, and they focus on different aspects of instability. First, their main ingredient is reinforcement learning: That is, they assume traders choose their beliefs according to a fitness measure, such as past profits made by the belief. In contrast, my analysis applies regardless of how beliefs (and disagreements) are formed. Second, they focus on prices and show that financial innovation combined with reinforcement learning makes prices more likely to be dynamically unstable. In contrast, I focus on equilibrium portfolios; more specifically, my notion of instability is an increase in traders' portfolio risks.

The hedge-more/bet-more effect also appears in Dow (1998), who analyzes financial innovation in the context of market liquidity with asymmetric information. He considers the introduction of a new asset that makes arbitrage less risky. In view of the hedge-more/bet-more effect, this induces arbitrageurs to trade more aggressively. The main result is that more aggressive arbitrage could then make everyone worse off by exacerbating adverse selection. In contrast, I analyze the effect of the hedge-more/bet-more effect on portfolio risks, and I show that financial innovation can increase these risks even in the absence of informational channels.

Other closely related papers include Weyl (2007) and Dieckmann (2009), which emphasize that increased trading opportunities might increase portfolio risks when traders have distorted or different beliefs.[4] Weyl (2007) notes that cross-market arbitrage might create risks when investors have mistaken beliefs. Dieckmann (2009) shows that rare-event insurance can increase portfolio risks when traders disagree about the frequency of these events. The contribution of my paper is to systematically characterize the effect of financial innovation on portfolio risks for a general environment with belief disagreements and mean-variance preferences. I also analyze endogenous financial innovation and show that it is partly driven by the speculation motive for trade. In recent work, Shen, Yan, and Zhang (2012) emphasize that endogenous financial innovation will also be directed towards mitigating traders' collateral constraints (which I abstract away from).

Finally, there is a large finance literature which analyzes the implications of belief dis-

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[4]The potentially destabilizing role of speculation is also discussed in Stiglitz (1989), Summers and Summers (1991), and Stout (1995). Posner and Weyl (2012) push the normative implications further by calling for a regulatory authority which approves financial products based on whether they will increase or decrease the portfolio risks.

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agreements for asset prices or volume of trade. A very incomplete list includes Lintner (1969), Rubinstein (1974), Miller (1977), Harrison and Kreps (1978), Varian (1985, 1989), Harris and Raviv (1993), Kandel and Pearson (1995), Zapatero (1998), Chen, Hong and Stein (2003), Scheinkman and Xiong (2003), Geanakoplos (2009), Cao (2010), Simsek (2012), Kubler and Schmedders (2012). The main difference from this literature is the focus on the effect of belief disagreements on *the riskiness of traders' portfolios*, rather than the volatility (and the level) of prices or the volume of trade.

The rest of the paper is organized as follows. Section 2 introduces the basic environment. This section also uses simple examples to illustrate the two channels by which new assets increase traders' portfolio risks. Section 3 characterizes the equilibrium and decomposes traders' portfolio risks into the uninsurable and the speculative variance components. Section 4 presents the main result, which characterizes the effect of financial innovation on these two components of portfolio risks. This section also relates the changes in portfolio risks to empirical measures such as trading volume. Section 5 analyzes endogenous financial innovation. Section 6 discusses the welfare implications and Section 7 concludes. Appendix A contains the results and proofs omitted from the main text.

2 Basic Environment and Main Channels

Consider an economy with two dates, $\{0, 1\}$, and a single consumption good, which will be referred to as a dollar. There are a finite number of traders denoted by $i \in I = \{1, 2, .., |I|\}$. Each trader is endowed with e dollars at date 0, which is constant. Trader i is also endowed with w_i dollars at date 1, which is a random variable that captures the trader's background risks. Traders only consume at date 1, and they can transfer resources to date 1 by investing in one of two ways. They can invest in cash which yields one dollar for each dollar invested. Alternatively, they can invest in risky assets denoted by $j \in J = \{1, .., |J|\}$. Asset j is in fixed supply, normalized to zero, and it pays a^j dollars at date 1, which is a random variable. Assets' payoffs and prices are respectively denoted by $|J| \times 1$ column vectors $\mathbf{a} = \left(a^1, .., a^{|J|}\right)'$.

The uncertainty in this economy is captured by an $|m| \times 1$ random vector, $\mathbf{v} = (v_1, .., v_m)'$. Traders' date 1 endowments and asset payoffs can be written as linear combinations of $\mathbf{v}$:

$$w_i = (\mathbf{W}_i)' \mathbf{v} \text{ and } a^j = \left(\mathbf{A}^j\right)' \mathbf{v}, \text{ for each } i \in I \text{ and } j \in J,$$

where $\mathbf{W}_i \in \mathbb{R}^m$ and $\mathbf{A}^j$ are $|m| \times 1$ vectors. The vectors, $\left\{\mathbf{A}^j\right\}_j$, are linearly independent, which ensures that assets are not redundant. These assets can be directly thought of as futures whose payoffs are also linear functions of their underlying assets. However, the economic insights generalize to non-linear derivatives (such as options) and other exotic new assets. The following is the key assumption I make about traders' beliefs.

Assumption (A1). Trader i's prior belief for $\mathbf{v}$ has a Normal distribution, $N\left(\boldsymbol{\mu}_i^{\mathbf{v}}, \Lambda^{\mathbf{v}}\right)$, where $\boldsymbol{\mu}_i^{\mathbf{v}} \in \mathbb{R}^m$ is the mean vector and $\Lambda^{\mathbf{v}}$ is the $m \times m$ covariance matrix with full row rank. Traders

agree to disagree in the sense that each trader knows all other traders' beliefs.

The first part of the assumption says that traders can potentially have different beliefs about the mean of the underlying uncertainty, $\mathbf{v}$. Traders are also assumed to agree on the variance. This feature ensures closed form solutions but otherwise does not play an important role. The important ingredient is that traders have different beliefs about asset values; whether these differences come from variances or means is not central.[5] The second part of assumption (A1) says that belief differences do not correspond to traders' private information, but rather, they correspond to *disagreements.* This assumption ensures that traders will actually trade on their different beliefs, circumventing the well known no-trade theorems (e.g., Milgrom and Stokey, 1982). A recent and growing literature suggests that belief disagreements are important to explain various aspects of financial markets (see Hong and Stein, 2007, for a review). The purpose of this paper is to analyze the implications of disagreements for financial innovation.

At date 0, traders take positions in risky assets in a competitive market. Let p^j denote the price of asset j, and $\mathbf{p} = \left(p^1, .., p^{|J|}\right)'$ denote the $|J| \times 1$ price vector. Trader i's position in risky assets is denoted by the vector, $\mathbf{x}_i = \left(x_i^1, .., x_i^{|J|}\right)'$, where $x_i^j \in \mathbb{R}$. The trader invests the rest of her initial endowment, $e - \mathbf{x}_i'\mathbf{p} \in \mathbb{R}$, in cash.[6] With these investment decisions, her net worth at date 1 is given by:

$$n_i = e - \mathbf{x}_i'\mathbf{p} + w_i + \mathbf{x}_i'\mathbf{a}. \tag{1}$$

Trader i maximizes subjective expected utility over net worth at date 1. Her utility function takes the CARA form. Since the asset payoffs and endowment shocks are jointly Normally distributed, the trader's optimization reduces to the usual mean-variance problem:[7]

$$\max_{\mathbf{x}_i} E_i\left[n_i\right] - \frac{\theta_i}{2} var_i\left[n_i\right]. \tag{2}$$

Here, θ_i denotes the trader's absolute risk aversion coefficient, while $E_i\left[\cdot\right]$ and $var_i\left[\cdot\right]$ respectively denote the mean and the variance of the trader's portfolio according to her beliefs.

The equilibrium in this economy is a collection of asset prices, $\mathbf{p}$, and portfolios, $\left(\mathbf{x}_1, .., \mathbf{x}_{|I|}\right)$, such that each trader i chooses her portfolio to solve problem (2) and prices clear asset markets,

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[5]In a continuous time setting with Brownian motion, Bayesian learning would immediately reveal the objective volatility of the underlying process. In contrast, the mean is much more difficult to learn, lending some additional credibility to assumption (A1). In view of this observation, the common belief for the variance, $\Lambda^{\mathbf{v}}$, can also be reasonably assumed to be the same as the objective variance of $\mathbf{v}$.

[6]Note that traders are allowed to take unrestricted negative positions in risky assets or cash, that is, both short selling and leverage are allowed. Similarly, the asset payoffs can take negative values because the environment is frictionless. In particular, there is no limited liability and repayment is enforced by contracts.

[7]The only role of the CARA preferences and the Normality assumption is to generate the mean-variance optimization in (2). In particular, the results of this paper apply as long as traders' portfolio choice can be reduced to the form in (2) over net worth. An important special case is the continuous-time model in which traders have time-separable expected utility preferences (which are not necessarily CARA), and asset returns and background risks follow diffusion processes. In this case, the optimization problem of a trader at any date can be reduced to the form in (2) (see Ingersoll, 1987). The only caveat is that the reduced form coefficient of absolute risk aversion, θ_i, is endogenous since it depends on the trader's value function. Thus, in the continuous trading environment, the results of this paper apply at a trading date conditional on traders' coefficients of absolute risk aversion, $\{\theta_i\}_i$.

that is, $\sum_i x_i^j = 0$ for each $j \in J$. I will capture financial innovation in this economy as an expansion of the set of traded assets. Before I turn to the general characterization, I use a simple example to illustrate the effects of financial innovation on portfolio risks.

2.1 An illustrative example

Suppose there are two traders with the same coefficient of risk aversion, i.e., $I = \{1, 2\}$ and $\theta_1 = \theta_2 \equiv \theta$. The underlying uncertainty is captured by two uncorrelated random variables, v_1, v_2. Traders' background risks depend on a combination of the two random variables. Moreover, they are perfectly negatively correlated with one another, that is:

$$w_1 = v \text{ and } w_2 = -v, \text{ where } v = v_1 + \alpha v_2.$$

As a benchmark suppose traders have common beliefs about v_1 and v_2 given by $N(0, 1)$. In this benchmark, first consider the case in which there are no assets, i.e., $J = \emptyset$. In this case, there is no trade and traders' net worths are given by:

$$n_1 = e + v \text{ and } n_2 = e - v. \tag{3}$$

Traders' net worths are risky because they are unable to hedge their endowment risks. Next suppose a new asset is introduced whose payoff is perfectly correlated with traders' endowments,

$$a^1 = v = v_1 + \alpha v_2.$$

In this case, traders' equilibrium portfolios are given by:

$$x_1^1 = -1 \text{ and } x_2^1 = 1$$

(and the equilibrium price is $p^1 = 0$). Traders' net worths are constant and given by

$$n_1 = n_2 = e.$$

Thus, the benchmark analysis shows that, with common beliefs, financial innovation enables traders to hedge and diversify their idiosyncratic risks.

Next suppose traders have belief disagreements about some of the uncertainty in this economy. In particular, traders have common beliefs for v_2 given by the distribution, $N(0, 1)$. They also know that v_1 and v_2 are uncorrelated. However, they disagree about the distribution of v_1. Trader 1's prior belief for v_1 is given by $N(\varepsilon, 1)$ while trader 2's belief is given by $N(-\varepsilon, 1)$. The parameter ε captures the level of the disagreement. I next use this specification to illustrate the two channels by which new assets increase portfolio risks.

Channel 1: New assets generate new disagreements

Consider the case in which asset 1 is available for trade. Since traders disagree about the mean of v_1, they also disagree about the mean of the asset payoff, $a^1 = v_1 + \alpha v_2$. In this case, it is easy to check that the asset price is $p^1 = 0$ (by symmetry) and that traders' portfolios are:

$$x_1^1 = -1 + x_1^S \text{ and } x_2^1 = 1 + x_2^S, \tag{4}$$
$$\text{where } x_1^S = \frac{\varepsilon}{\theta}\frac{1}{1+\alpha^2} \text{ and } x_2^S = -\frac{\varepsilon}{\theta}\frac{1}{1+\alpha^2}.$$

Note that traders' positions deviate from the optimal risk sharing benchmark in view of their disagreements. I define the difference as the traders' *speculative portfolios* and denote it by $\{x_i^S\}_i$. Traders' net worths can also be calculated as:

$$n_1 = e + \frac{\varepsilon}{\theta}\frac{v_1 + \alpha v_2}{1+\alpha^2} \text{ and } n_2 = e - \frac{\varepsilon}{\theta}\frac{v_1 + \alpha v_2}{1+\alpha^2}. \tag{5}$$

If $\varepsilon > \theta\left(1+\alpha^2\right)$, then traders' net worths are riskier than the case in which no new asset is introduced [cf. Eq. (3)]. Intuitively, trader 1 is so optimistic about the asset's payoff that she takes a positive net position, despite the fact that her endowment covaries positively with the asset payoff. Consequently, the new asset increases the riskiness of her net worth. Hence, when traders' disagreements about the asset payoff are sufficiently large, financial innovation increases traders' portfolio risks.

Channel 2: New assets amplify speculation on existing disagreements

Next consider the introduction of a second asset with payoff, $a^2 = v_2$. Note that traders do not disagree on the payoff of this asset. Nonetheless, this asset also increases traders' portfolio risks through a second channel: By amplifying traders' speculation on existing disagreements. To see this, first consider traders' equilibrium portfolios in this case which can be calculated as:

$$\begin{bmatrix} x_1^1 \\ x_1^2 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + \begin{bmatrix} x_1^{1,S} \equiv \frac{\varepsilon}{\theta} \\ x_1^{2,S} \equiv -\alpha\frac{\varepsilon}{\theta} \end{bmatrix} \text{ and } \begin{bmatrix} x_2^1 \\ x_2^2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} x_2^{1,S} \equiv -\frac{\varepsilon}{\theta} \\ x_2^{2,S} \equiv \alpha\frac{\varepsilon}{\theta} \end{bmatrix}, \tag{6}$$

As before, traders' portfolios feature speculative positions, $\left\{ \begin{bmatrix} x_i^{1,S} \\ x_i^{2,S} \end{bmatrix} \right\}_i$, which represent the deviations from the optimal risk sharing benchmark. Given these positions, traders' net worths are given by:

$$n_1 = e + \frac{\varepsilon}{\theta}v_1 \text{ and } n_2 = e - \frac{\varepsilon}{\theta}v_1. \tag{7}$$

Note that the magnitude of traders' speculative positions on asset 1 is greater than the earlier setting in which asset 2 was not available [cf. Eqs. (6) and (4)]. Importantly, traders' net worths are also riskier [cf. Eqs. (7) and (5)]. Put differently, the innovation of asset 2, about

which traders do not disagree, enables traders to take greater speculative positions on asset 1 and increases their portfolio risks.

The intuition for this result is related to an important economic force: the *hedge-more/bet-more* effect. When only asset 1 is available, traders' speculative positions and portfolio risks are decreasing in α, the share of v_2 in asset 1's payoff [cf. Eqs. (4) and (5)]. Intuitively, asset 1 provides the traders with only an *impure bet* because its payoff also depends on the risk, v_2, on which traders do not disagree. To take speculative positions, traders must also hold these additional risks, which makes them reluctant to bet. When asset 2 is also available, traders complement their speculative positions in asset 1 by taking the opposite positions in asset 2 [Eq. (6)]. This enables them to take a purer bet on the risk, v_1. When traders are able to take purer bets, they also take larger bets, which in turn leads to greater portfolio risks [Eq. (7)].

3   Equilibrium and the Decomposition of Average Variance

This section characterizes the equilibrium, and decomposes traders' portfolio risks into two components which respectively correspond to traders' risk sharing and speculative motives for trade. The main result in the next section characterizes the effect of financial innovation on the two components of portfolio risks.

Given assumption (A1), trader i believes the asset payoffs are Normally distributed, $N\left(\boldsymbol{\mu}_i, \Lambda\right)$, with

$$\boldsymbol{\mu}_i \equiv \mathbf{A}' \boldsymbol{\mu}_i^{\mathbf{v}} \text{ and } \Lambda \equiv \mathbf{A}' \Lambda^{\mathbf{v}} \mathbf{A},$$

where $\boldsymbol{\mu}_i$ is a $|J| \times 1$ vector and Λ is a $|J| \times |J|$ matrix. In addition, trader i believes that the covariance of her endowment with the asset payoffs is given by:

$$\boldsymbol{\lambda}_i = \mathbf{A}' \Lambda^{\mathbf{v}} \mathbf{W}_i,$$

where $\boldsymbol{\lambda}_i$ is a $|J| \times 1$ matrix. Given these beliefs, traders' portfolio demand [cf. problem (2)] can be solved in closed form. Aggregating traders' demands and using market clearing, asset prices are given by:

$$\mathbf{p} = \frac{1}{|I|} \sum_{i \in I} \left(\frac{\bar{\theta}}{\theta_i} \boldsymbol{\mu}_i - \bar{\theta} \boldsymbol{\lambda}_i \right), \tag{8}$$

where $\bar{\theta} \equiv \left(\sum_{i \in I} \theta^{-1} / |I| \right)^{-1}$ is the Harmonic mean of traders' absolute risk aversion coefficients. Intuitively, an asset commands a higher price if traders are on average optimistic about its payoff, or if it on average covaries negatively with traders' endowments.

Using the prices in (8), a trader's equilibrium portfolio can also be solved in closed form:

$$\mathbf{x}_i = \mathbf{x}_i^R + \mathbf{x}_i^S, \quad \text{where} \tag{9}$$

$$\mathbf{x}_i^R = -\Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i \quad \text{and} \quad \mathbf{x}_i^S = \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i}.$$

Here, the expression

$$\tilde{\boldsymbol{\lambda}}_i = \boldsymbol{\lambda}_i - \frac{\bar{\theta}}{\theta_i} \frac{1}{|I|} \sum_{\bar{i} \in I} \boldsymbol{\lambda}_{\bar{i}} \tag{10}$$

denotes the relative covariance of the trader's endowment, and

$$\tilde{\boldsymbol{\mu}}_i = \boldsymbol{\mu}_i - \frac{1}{|I|} \sum_{\bar{i} \in I} \frac{\bar{\theta}}{\theta_{\bar{i}}} \mu_{\bar{i}} \tag{11}$$

denotes her relative optimism. Note that the trader's portfolio has two components. The first component, $\mathbf{x}_i^R$, is the portfolio that would obtain if there were no belief belief disagreements (i.e., if $\tilde{\boldsymbol{\mu}}_i = 0$ for each i). Hence, I refer to $\mathbf{x}_i^R$ as the trader's *risk sharing portfolio*. The optimal risk sharing portfolio is determined by traders' endowment risks and their risk tolerances. The second component, $\mathbf{x}_i^S$, captures traders' deviations from this benchmark in view of their belief disagreements. Hence, I refer to $\mathbf{x}_i^S$ as the *speculative portfolio* of trader i.

Eqs. $(8) - (11)$ complete the characterization of equilibrium in this economy. The main goal of this paper is to analyze the effect of financial innovation on portfolio risks. Given the mean-variance framework, a natural measure of portfolio risk for a trader i is the variance of her net worth. I consider an average of this measure across all traders, *the average variance*, defined as follows:

$$\Omega = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} var\left(n_i\right) = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \left(\mathbf{W}_i' \Lambda^{\mathbf{v}} \mathbf{W}_i + 2\mathbf{x}_i' \boldsymbol{\lambda}_i + \mathbf{x}_i' \Lambda \mathbf{x}_i\right). \tag{12}$$

A couple of comments about this definition are in order. First, the portfolio risk of a trader is calculated according to traders' (common) belief for the variance $\Lambda^{\mathbf{v}}$. Second, traders that are relatively more risk averse are given a greater weight in the average.

I use Ω as my main measure of average portfolio risks for two reasons. First, Section 6 shows that Ω is a natural measure of welfare in this economy when traders' welfare is calculated according to any common belief (as opposed to their own beliefs). The second justification is provided by the following lemma.

Lemma 1. *The risk sharing portfolios, $\{\mathbf{x}_i^R\}_i$, minimize the average variance, Ω, among all feasible portfolios:*

$$\min_{\{\mathbf{x}_i \in \mathbb{R}^{|J|}\}_i} \Omega \qquad s.t. \sum_i \mathbf{x}_i = 0. \tag{13}$$

When there are no belief disagreements, i.e., $\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}} = 0$ for each i, then the complete portfolios and the risk sharing portfolios coincide, i.e., $\mathbf{x}_i = \mathbf{x}_i^R$ for each i. Thus, Lemma 1 shows that Ω is the measure of risks that would be minimized in equilibrium absent belief disagreements. Thus, it is natural to take Ω as the measure of average risks, and to characterize the extent to which it deviates from the minimum benchmark in (13) when traders have belief disagreements.

To this end, I let Ω^R denote the minimum value of problem (13) and refer to it as the *uninsurable variance*. I also define $\Omega^S = \Omega - \Omega^R$ and refer to it as the *speculative variance*.

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This provides a decomposition of the average variance into two components:

$$\Omega = \quad \Omega^R \quad + \quad \Omega^S.$$

The main result in the next section concerns the effect of financial innovation on Ω^R and Ω^S. The next lemma characterizes the two components of average variance in terms of the exogenous parameters of the model. The forms of Ω^R and Ω^S are intuitive. The uninsurable variance is lower when the assets provide better risk sharing opportunities, captured by larger $\tilde{\boldsymbol{\lambda}}_i$, whereas, the speculative variance is greater when the assets feature greater belief disagreements, captured by larger $\tilde{\boldsymbol{\mu}}_i$.

Lemma 2. *The uninsurable variance is given by:*

$$\Omega^R = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \left(\mathbf{W}_i' \Lambda^{\mathbf{v}} \mathbf{W}_i - \tilde{\boldsymbol{\lambda}}_i' \Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i \right), \tag{14}$$

and the speculative variance is given by:

$$\Omega^S \equiv \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i}' \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} \right). \tag{15}$$

4   Financial Innovation and Portfolio Risks

I model financial innovation as an expansion of the set of traded assets. For this purpose, it is useful to define the notation, $z\left(\hat{J}\right)$, to refer to the equilibrium variable z when only a subset $\hat{J} \subset J$ of the assets in J are traded. I next present the main result.

Theorem 1 (Financial Innovation and Portfolio Risks). *Suppose J consists of a set of old and new assets, J_O and J_N (formally, $J = J_O \cup J_N$ where J_O and J_N are disjoint sets).*
(i) Financial innovation always reduces the uninsurable variance, that is:

$$\Delta \Omega^R = \Omega^R \left(J_O \cup J_N \right) - \Omega^R \left(J_O \right) \leq 0.$$

(ii) Financial innovation always increases the speculative variance, that is:

$$\Delta \Omega^S = \Omega^S \left(J_O \cup J_N \right) - \Omega^S \left(J_O \right) \geq 0.$$

The first part of this theorem is a corollary of Lemma 1, and it shows that financial innovation always provides some risk sharing benefits. This part formalizes the traditional view of financial innovation in the context of this model. On the other hand, the second part of the theorem identifies a second force which always operates in the opposite direction. In particular, when there are belief disagreements, financial innovation also always increases the speculative variance. Hence, the net effect of financial innovation on average variance is

ambiguous, and it depends on the relative strength of the two forces. It is easy to see that, when belief disagreements are sufficiently large, the effect on speculative variance is sufficiently strong that financial innovation increases the average variance.

Most of the literature on financial innovation considers the special case without belief disagreements. Theorem 1 shows that the common beliefs assumption is restrictive, as it shuts down an important economic force by which financial innovation has a positive effect on portfolio risks. It is also worth emphasizing the generality of Theorem 1. The result applies for all sets of existing and new assets, J_O and J_N, with no restrictions on the joint distribution of asset payoffs or traders' beliefs for $\mathbf{v}$ [except for the relatively mild Assumption (A1)]. For example, Theorem 1 shows that financial innovation increases the speculative variance even if there are no belief disagreements about new assets (as in Example 1).

The rest of this section provides a sketch proof for the second part of Theorem 1, which is useful to develop the intuition for the result. Consider an economy which is identical to the original economy except that there are no background risks (i.e., $\mathbf{W}_i = 0$ for all $i \in I$), so that the only motive for trade is speculation. The proof in the appendix shows that the average variance in this economy is identical to the speculative variance in the original economy. Thus, it suffices to show that financial innovation increases average portfolio risks in the hypothetical economy.

Recall that the Sharpe ratio of a portfolio is defined as the expected portfolio return in excess of the risk-free rate (which is normalized to 0) divided by the standard deviation of the portfolio return. Traders in the hypothetical economy perceive positive Sharpe ratios because they think various assets are mispriced. Define a trader's *speculative Sharpe ratio* as the Sharpe ratio of her equilibrium portfolio. Using Eqs. $(8) - (11)$, this can be calculated as:

$$Sharpe_i^S = \frac{\left(\mathbf{x}_i^S\right)' (\boldsymbol{\mu}_i - \mathbf{p})}{\sqrt{\left(\mathbf{x}_i^S\right)' \Lambda \mathbf{x}_i^S}} = \sqrt{\tilde{\boldsymbol{\mu}}_i' \Lambda^{-1} \tilde{\boldsymbol{\mu}}_i}. \tag{16}$$

Next consider the trader's *portfolio return* given by n_i/e (where recall that e is the trader's initial net worth). The standard deviation of this return can also be calculated as:

$$\sigma_i^S \equiv \frac{1}{e} \sqrt{\left(\mathbf{x}_i^S\right)' \Lambda \mathbf{x}_i^S} = \frac{1}{\theta_i e} \sqrt{\tilde{\boldsymbol{\mu}}_i' \Lambda^{-1} \tilde{\boldsymbol{\mu}}_i}. \tag{17}$$

Note that the ratio, $\theta_i e$, provides a measure of trader i's coefficient of relative risk aversion. Thus, combining Eqs. (16) and (17) gives the familiar result that the standard deviation of the portfolio return is equal to the Sharpe ratio of the optimal portfolio divided by the coefficient of relative risk aversion (see Campbell and Viceira, 2002). Intuitively, if a trader finds a risky portfolio with a higher Sharpe ratio, then she exploits this opportunity to such an extent that she ends up with greater portfolio risks. This textbook result also applies in this model for the hypothetical economy with no background risks.

Theorem 1 can then be understood from the lenses of this textbook result. Financial

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innovation increases each trader's speculative Sharpe ratio (as formally demonstrated in the appendix) because it expands traders' *betting possibilities frontier.* That is, when the asset set is $\hat{J} = J^O \cup J^N$, traders are able to make all the speculative trades they could make when the asset set is $\hat{J} = J^O$, and some more. Importantly, new assets expand the betting possibilities frontier through the two channels emphasized before. First, new assets generate new disagreements, which creates higher expected returns (thereby increasing the numerator of the speculative Sharpe ratio). Second, new assets also enable each trader to take purer bets on existing disagreements (thereby reducing the denominator of the speculative Sharpe ratio). Once a trader obtains a higher speculative Sharpe ratio, she also undertakes greater speculative risks, providing a sketch proof for the main result.

4.1 Assessing the net effect of financial innovation

In view of Theorem 1, the net effect of financial innovation on portfolio risks depends on the relative strength of the changes in the two components, $|\Delta\Omega^S|$ and $|\Delta\Omega^R|$. A number of comparative statics for the changes, $|\Delta\Omega^S|$ and $|\Delta\Omega^R|$, can be obtained by inspecting the functional forms in Eqs. (15) and (14). Increasing traders' belief disagreements (by scaling up $\{\boldsymbol{\mu}_i\}_i$ proportionally) or decreasing their risk aversion coefficients (by scaling down $\{\theta_i\}$) increase $|\Delta\Omega^S|$ without affecting $|\Delta\Omega^R|$. Intuitively, the speculation motive is stronger relative to the risk sharing motive when traders have greater belief disagreements, or when they are less risk averse. Therefore, these environments are particularly susceptible to an increase in portfolio risks in response to financial innovation.

To make more precise statements, the changes in portfolio risks, $|\Delta\Omega^S|$ and $|\Delta\Omega^R|$, need to be empirically measured. The following result takes a step in this direction by linking portfolio risks to traders' speculative and risk sharing positions.

Proposition 1 (Traders' Positions and Portfolio Risks). *Let* $\left(\mathbf{x}_i^S\left(J_O\right), \mathbf{x}_i^R\left(J_O\right)\right)$ *denote traders' portfolios before financial innovation and* $\Lambda\left(J_O\right)$ *denote the variance matrix for the old assets,* J_O. *Then, the effect of financial innovation on portfolio risks,* $|\Delta\Omega^S|$ *and* $|\Delta\Omega^R|$, *can be written as:*

$$
\begin{aligned}
\left|\Delta\Omega^S\right| &= \frac{1}{|I|}\sum_{i\in I}\frac{\theta_i}{\overline{\theta}}\left(\mathbf{x}_i^{S\prime}\Lambda\mathbf{x}_i^S - \mathbf{x}_i^S\left(J_O\right)'\Lambda\left(J_O\right)\mathbf{x}_i^S\left(J_O\right)\right), &(18)\\
\left|\Delta\Omega^R\right| &= \frac{1}{|I|}\sum_{i\in I}\frac{\theta_i}{\overline{\theta}}\left(\mathbf{x}_i^{R\prime}\Lambda\mathbf{x}_i^R - \mathbf{x}_i^R\left(J_O\right)'\Lambda\left(J_O\right)\mathbf{x}_i^R\left(J_O\right)\right).
\end{aligned}
$$

Intuitively, the two components of traders' portfolios, $\left(\mathbf{x}_i^S, \mathbf{x}_i^R\right)$, describe the extent to which the assets are used for the corresponding motive for trade. Hence, after an appropriate normalization, the changes in these components also describe the changes in portfolio risks as formalized in Proposition 1. To be empirically useful, this result requires separate identification of the speculative and the risk sharing portfolios, $\left(\mathbf{x}_i^S, \mathbf{x}_i^R\right)$. While this is no easy task, it is

facilitated by the fact that traders' actual portfolios, $\{\mathbf{x}_i\}_i$, are in principle observable (e.g., by regulators). Hence, it might suffice to estimate, or at the very least to bound, traders' risk sharing portfolios, $\{\mathbf{x}_i^R\}_i$. This requires some knowledge of traders' background risks, but no knowledge of their beliefs or the assets' expected returns [cf. Eq. (9)]. Traders' speculative portfolios can then be identified as deviations from the risk-sharing portfolios, $\mathbf{x}_i^S = \mathbf{x}_i - \mathbf{x}_i^R$. Alternatively, traders' beliefs can also be measured directly either by using survey data (see, Greenwood and Shleifer (2012) for a recent application) or by considering their actions in related economic problems (see, Cheng, Raina, and Xiong (2012) for an application in the context of the recent crisis). The measured beliefs can then be used to calibrate traders' speculative portfolios, $\{\mathbf{x}_i^S\}_i$.[8]

In some situations, it might be easier to observe aggregated measures of positions such as trading volume as opposed to traders' individual positions. Eqs. (18) suggest that trading volume could also be informative about portfolio risks. The next result establishes that the relationship is in fact quite tight in some special cases. To state the result, consider a single asset that is introduced for trade, that is, suppose $J_N = \{j_N\}$ for some j_N. Define respectively the *speculative and risk sharing trading volumes* in the new asset as:

$$T^{j_N,S} = \left(\frac{1}{|I|} \sum_i \frac{\theta_i}{\overline{\theta}} \left(x_i^{j_N,S}\right)^2\right)^{1/2} \text{ and } T^{j_N,R} = \left(\frac{1}{|I|} \sum_i \frac{\theta_i}{\overline{\theta}} \left(x_i^{j_N,R}\right)^2\right)^{1/2}.$$

Note that trading volume is defined to be a weighted (and quadratic) average of traders' positions, with a greater weight given to traders that are more risk averse.

Proposition 2 (Trading Volume and Portfolio Risks). *Suppose that a single asset, j_N, is introduced for trade, and that this asset's payoff is uncorrelated with all existing assets (that is, $\left(\mathbf{A}^j\right)' \Lambda^{\mathbf{v}} \mathbf{A}^{j_N} = 0$ for all $j \in J_O$). Then, financial innovation increases the average variance, that is, $\Omega(J) > \Omega(J_O)$, if and only if $T^{S,j_N} > T^{R,j_N}$, that is, the new asset leads to a greater speculative trading volume than risk-sharing trading volume.*

Intuitively, the speculative and the risk sharing trading volumes each describe the extent to which the new asset is used for the corresponding motive for trade. When a new asset is uncorrelated with existing assets, trading volume perfectly maps into portfolio risks as formalized by Proposition 2. In practice, it might be difficult to measure speculative and risk-sharing trading volume separately. As in Proposition 1, one possibility is to estimate or bound the risk-sharing volume, T^{R,j_N}, and to obtain the speculative trading volume, $T^{j_N,S}$, as the residual.[9]

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[8] I illustrate this approach further in a recent working paper version, Simsek (2011). There, I use survey data on expectations of GDP growth to calibrate belief disagreements in the context of the national income markets proposed by Athanasoulis and Shiller (2001).

[9] Proposition 2 also requires the new asset to be uncorrelated with existing assets, which is arguably restrictive. Without this assumption, trading volume does not fully summarize the effect on portfolio risks since the correlations between the positions on old and new assets also matter. Nonetheless, trading volume often provides a useful diagnostic tool even in these more general cases. To see this, consider Example 2.1 in which a

5 Endogenous Financial Innovation

The analysis so far has taken the set of new assets as exogenous. In practice, many financial products are introduced endogenously by economic agents with profit incentives. A large literature emphasizes risk sharing as a major driving force for endogenous financial innovation [e.g., Allen and Gale (1994), Duffie and Rahi (1995), Athanasoulis and Shiller (2000, 2001)].[10] A natural question, in view of the results in the earlier sections, is to what extent the risk sharing motive for financial innovation is robust to the presence of belief disagreements. To address this question, this section endogenizes the asset design by introducing a profit seeking market maker and obtains two main results. First, the optimal asset design depends on the size and the nature of traders' belief disagreements, in addition to the possibilities for risk sharing. Second, when traders' belief disagreements are sufficiently large, the market maker designs assets that *maximize* traders' average portfolio risks among all possible choices, completely disregarding the risk sharing motive for financial innovation.

The main feature of the model in this section is that the assets, J, are introduced by a market maker. The market maker is constrained to choose $|J| \leq m$ assets, but is otherwise free to choose the asset design, $\mathbf{A}$. Here, recall that the matrix, $\mathbf{A} = \left[\mathbf{A}^1, \mathbf{A}^2, .., \mathbf{A}^{|J|} \right]$, captures the asset payoffs which are given by $a^j = \left(\mathbf{A}^j \right)' \mathbf{v}$ for each j. Thus, the market maker's choice of $\mathbf{A}$ affects the belief disagreements and the relative covariances according to [cf. Eqs. (11) and (10)]:

$$\tilde{\boldsymbol{\mu}}_i \left(\mathbf{A} \right) = \mathbf{A}' \tilde{\boldsymbol{\mu}}_i^{\mathbf{v}} \text{ and } \tilde{\boldsymbol{\lambda}}_i \left(\mathbf{A} \right) = \mathbf{A}' \Lambda^{\mathbf{v}} \tilde{\mathbf{W}}_i,$$

where the deviation terms are defined as:

$$\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}} = \boldsymbol{\mu}_i^{\mathbf{v}} - \frac{1}{|I|} \sum_{\bar{\imath} \in I} \frac{\bar{\theta}}{\theta_{\bar{\imath}}} \boldsymbol{\mu}_{\bar{\imath}}^{\mathbf{v}} \text{ and } \tilde{\mathbf{W}}_i = \mathbf{W}_i - \frac{\bar{\theta}}{\theta_i} \frac{1}{|I|} \sum_{\bar{\imath} \in I} \mathbf{W}_{\bar{\imath}}.$$

Once the market maker chooses the asset design, $\mathbf{A}$, the assets are traded in a competitive market similar to the previous sections. The market maker intermediates these trades which enables it to extract some of the surplus from traders. In practice, the market maker does so by charging commissions or bid-ask spreads. To keep the analysis simple, suppose the market maker in the model extracts the full surplus.[11] In particular, the market maker sets a fixed

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second asset, $a^2 = v_2$, that is correlated with the first asset is introduced. This asset generates some speculative trading volume, i.e., $x_i^{2,S} \neq 0$ for each i, and also leads to an increase in speculative variance, $\left| \Delta \Omega^S \right| > 0$. In contrast, the asset generates no risk-sharing trading volume, i.e., $x_i^{2,R} = 0$ for each i, and has no impact on the risk sharing variance, i.e., $\left| \Delta \Omega^R \right| = 0$.

[10] Risk sharing is one of several drivers of financial innovation emphasized by the previous literature. Other factors include mitigating agency frictions, reducing asymmetric information, minimizing transaction costs, and sidestepping taxes and regulation (see Tufano, 2004, for a recent survey). These other factors, while clearly important, are left out of the analysis in this paper to focus on the effect of belief disagreements on the risk sharing motive for innovation.

[11] The results below remain unchanged under the less extreme (reduced form) assumption that the market maker extracts a constant fraction, $\zeta \in (0, 1]$, of the surplus regardless of the choice of $\mathbf{A}$.

membership fee, π_i, for each trader i and makes a take it or leave it offer. If trader i accepts the offer, then she can trade the available assets in the competitive market. Otherwise, trader i is out of the market, and her net worth is given by her endowment, $e + \mathbf{W}_i'\mathbf{v}$.

In equilibrium, all traders accept the market maker's offer. Since traders have CARA preferences, their portfolio choices are independent of the fixed fees they pay. Hence, the equilibrium characterization is unchanged. Consider next the surplus the market maker extracts for a given choice of $\mathbf{A}$. If trader i were to reject the offer, she would receive the certainty equivalent payoff from her endowment. By accepting, she receives the certainty equivalent payoff from her equilibrium portfolio net of the fixed fee, $\pi_i(\mathbf{A})$. The market maker sets $\pi_i(\mathbf{A})$ so that the trader is just indifferent to accept. The analysis in the appendix shows that the market maker's total profits are given by:

$$\sum_{i \in I} \pi_i(\mathbf{A}) = \sum_i \frac{\theta_i}{2} \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A}) \right)' \Lambda^{-1} \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A}) \right). \tag{19}$$

This expression reflects the two motives for trade in this economy. Traders are willing to pay to trade assets that facilitate better risk sharing [i.e., larger $\tilde{\boldsymbol{\lambda}}_i(\mathbf{A})$], or those that generate greater belief disagreements [i.e., larger $\tilde{\boldsymbol{\mu}}_i(\mathbf{A})$].

The market maker chooses an asset design, $\mathbf{A}$, that maximizes the expected profits in (19). Note that many choices of $\mathbf{A}$ represent the same trading opportunities over the space of the underlying risks, $\mathbf{v}$ (and thus, also generate the same profits). Thus, suppose without loss of generality that the market maker's choice is subject to the following normalizations:

$$\Lambda = \mathbf{A}'\Lambda^{\mathbf{v}}\mathbf{A} = \mathbf{I}_{|J|}, \qquad \text{and} \qquad \left((\Lambda^{\mathbf{v}})^{1/2} \mathbf{A} \right)_1^j \geq 0 \text{ for each } j \in J. \tag{20}$$

Here, $(\Lambda^{\mathbf{v}})^{1/2}$ denotes the unique positive definite square root of the matrix, $\Lambda^{\mathbf{v}}$. The first condition in (20) normalizes the variance of assets to be the identity matrix, $\mathbf{I}_{|J|}$. This condition determines the column vectors of the matrix, $(\Lambda^{\mathbf{v}})^{1/2} \mathbf{A}$, up to a sign. The second condition resolves the remaining indeterminacy by adopting a sign convention for these column vectors.

Proposition 3 (Optimal Asset Design). *Suppose the matrix*

$$\frac{1}{|I|} \sum_i \frac{\theta_i}{\bar{\theta}} \left((\Lambda^{\mathbf{v}})^{-1/2} \frac{\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2} \tilde{\mathbf{W}}_i \right) \left((\Lambda^{\mathbf{v}})^{-1/2} \frac{\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2} \tilde{\mathbf{W}}_i \right)' \tag{21}$$

is non-singular. Then, an asset design is optimal if and only if the columns of the matrix for normalized asset payoffs, $(\Lambda^{\mathbf{v}})^{1/2} \mathbf{A}$, correspond to the eigenvectors corresponding to the $|J|$ largest eigenvalues of the matrix in (21). If the eigenvalues are distinct, then the asset design is uniquely determined by this condition along with the normalizations in (20). Otherwise, the asset design is determined up to a choice of the $|J|$ largest eigenvalues.

This result generalizes the results in Demange and Laroque (1995) and Athanasoulis and Shiller (2000) to the case with belief disagreements, $\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}} \neq \mathbf{0}$. Importantly, the expressions (19)

and (21) show that financial innovation is partly driven by the size and the nature of traders' belief disagreements. The size of the belief disagreements, $\left\lVert (\Lambda^{\mathbf{v}})^{-1/2} \tilde{\boldsymbol{\mu}}_i^{\mathbf{v}} \right\rVert$, (along with the risk aversion coefficients, θ_i) determine to what extent endogenous innovation is driven by the speculation motive for trade as opposed to risk sharing. When this term is significant, the nature of the belief disagreements, $\frac{(\Lambda^{\mathbf{v}})^{-1/2} \tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{\left\lVert (\Lambda^{\mathbf{v}})^{-1/2} \tilde{\boldsymbol{\mu}}_i^{\mathbf{v}} \right\rVert}$, bias the choice of assets towards those that maximize the opportunities for speculation.

The next result characterizes the optimal asset design further in two extreme cases: when traders have common beliefs, and when their belief disagreements are very large.

Theorem 2 (Endogenous Innnovation and Portfolio Risks). *Consider a collection of economies which are identical except for beliefs given by $\boldsymbol{\mu}_{i,K}^{\mathbf{v}} = K\boldsymbol{\mu}_i^{\mathbf{v}}$ for all i, where $K \geq 0$ is a parameter that scales belief disagreements. Suppose the matrix in (21) is non-singular with distinct eigenvalues for each K.*[12] *Let $\Omega_K(\cdot)$ denote the average variance and $\mathbf{A}_K$ denote the optimal asset design (characterized in Proposition 3) for each K.*

(i) With no belief disagreements, i.e., $K = 0$, the market maker innovates assets that minimize *the average variance:*

$$\mathbf{A}_0 \in \arg\min_{\hat{\mathbf{A}}} \Omega_0\left(\hat{\mathbf{A}}\right) \text{ subject to } (20).$$

For the next part, suppose there exists at least two traders with different beliefs, i.e., $\boldsymbol{\mu}_i^{\mathbf{v}} \neq \boldsymbol{\mu}_{\bar{i}}^{\mathbf{v}}$ for some $i, \bar{i} \in I$. Let $\Omega_K(\emptyset)$ denote the average variance without any assets.

(ii) As $K \to \infty$, the market maker innovates assets that maximize *the average variance. In particular, the limit of the optimal asset design, $\lim_{K \to \infty} \mathbf{A}_K$, and the limit of the (scaled) average variance, $\lim_{K \to \infty} \frac{1}{K^2} \Omega_K\left(\hat{\mathbf{A}}\right)$ exist and are finite. Moreover, these limits satisfy:*

$$\lim_{K \to \infty} \mathbf{A}_K \in \arg\max_{\hat{\mathbf{A}}} \left(\lim_{K \to \infty} \frac{1}{K^2} \Omega_K\left(\hat{\mathbf{A}}\right) \right) \text{ subject to } (20). \qquad (22)$$

Without belief disagreements, the market maker innovates assets that minimize average portfolio risks in this economy, as illustrated by the first part of the theorem. The second part provides a sharp contrast to this traditional view. When traders' belief disagreements are large, the market maker innovates assets that *maximize* average portfolio risks, completely disregarding the risk sharing motive for innovation. Intuitively, in this case speculation becomes the main motive for trade. As this happens, the market maker maximizes its profits by providing the traders with assets that enable them to bet most precisely on their different beliefs. As a by-product, the market maker also maximizes traders' speculative variance. When belief disagreements are large, this is equivalent to maximizing traders' average variance because the uninsurable variance is small relative to the speculative variance.

Theorem 2 illustrates that belief disagreements change not only the effect of a given set of assets on portfolio risks, as emphasized by Theorem 1, but also the nature of financial

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[12] The assumption of distinct eigenvalues can be relaxed at the expense of additional notation.

innovation. In particular, profit seeking agents might introduce assets that will increase risks, as opposed to those that will facilitate risk sharing as emphasized in the previous literature (e.g., Allen and Gale, 1994). Among other things, this observation might provide an explanation for why most of the macro futures markets proposed by Shiller (1993) have not been adopted in practice, despite the fact that they are in principle very useful for risk sharing purposes.

6 Financial Innovation and Welfare

While Theorems 1 and 2 show that financial innovation may increase portfolio risks, they do not reach any welfare conclusions. In fact, it is easy to see that financial innovation in the baseline setting results in a Pareto improvement if traders' welfare is calculated according to their own beliefs. This is because each trader perceives a large expected return from her speculative positions in new assets, which justifies the additional risks that she is taking. This section analyzes two variants of the baseline setting in which this welfare conclusion might be reversed. The first setting concerns an interpretation of belief disagreements as emerging from psychological biases. In this case, the Pareto criterion is arguably not appropriate and financial innovation might be inefficient according to an alternative welfare criterion. The second setting concerns an interpretation of traders as financial intermediaries under government protection. In this case, traders' portfolio choices are associated with externalities and financial innovation might be inefficient even according to the Pareto criterion. Importantly, in both settings a measure of average portfolio risks play a central role in welfare analysis, providing some normative content to Theorems 1 and 2.

6.1 Inefficiencies Driven by Belief Distortions

Various economists have recognized that the standard Pareto criterion might lead to unappealing conclusions in environments with belief disagreements (see, for example, Dreze (1970), Stiglitz (1989), Mongin (1997), Gilboa, Samet, and Schmeidler (2004)). To illustrate this point, consider a true story between two prominent economic theorists, Bob Wilson and Joe Stiglitz.[13] One day in 1970s, Bob and Joe disagreed about the contents of the pillow on a sofa. Joe thought that the pillow had a natural down filling, while Bob thought that a synthetic filling was more likely. Suppose both Bob and Joe assigned probability 0.9 to their own view. Given their different views, they naturally decided to construct a bet: They would each put $100 dollar and the winner (whose view is correct) would take the total of $200. However, they could only determine the winner by tearing the pillow apart to find out the actual content. Bob and Joe agreed to share the cost of replacing the pillow ($50). After reflecting on the situation, Bob and Joe realized that it would be Pareto optimal to destroy the pillow. This is because they both perceived a net return of $55 from speculation ($80 expected return from the bet

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[13] See Kreps (2012, page 193) for more details on this story. I have slightly changed the details to simplify the analysis.

minus $25 cost of replacing the pillow). Even though the bet was Pareto optimal, it looked quite unattractive: There would be a money transfer from one party to another, nothing would be produced, and a perfectly good pillow would be destroyed.

Theorems 1 and 2 are conceptually similar to the story between Bob and Joe. When belief disagreements are large, these results show that financial innovation increases traders' average portfolio risks. Note that traders are risk averse, and therefore, are willing to pay to insure their risks. Therefore, the increase in their portfolio risks corresponds to (certainty equivalent) wealth destruction, which is analogous to the destruction of the pillow in the above example. The remaining question is whether Pareto optimality is the appropriate welfare criterion despite the wealth destruction that it leads to.

This question requires one to take a stance on the source of traders' belief disagreements. On the one hand, these disagreements might reflect traders' different subjective prior beliefs as in Savage (1954). Under this interpretation, traders' beliefs are convenient representations of their preferences under uncertainty. The fact that they have different beliefs is a reflection of their differing personal experiences. In this case, the Pareto criterion might be appropriate despite the fact that it leads to wealth destruction.[14] On the other hand, belief disagreements might also represent mistakes in interpretation of information. A large and growing literature in behavioral economics and finance has emphasized that individuals' beliefs are distorted in view of various psychological biases such as overconfidence, the representativeness heuristic, the conservativeness bias, limited attention and so on (see Barberis and Thaler (2003) for a survey). If individuals' beliefs are heterogeneously distorted, then they would naturally come to have belief disagreements. Under this interpretation, the Pareto criterion is arguably no longer appropriate, since individuals' welfare should be evaluated according to the objective (or non-distorted) belief. However, there is a practical difficulty because the planner might also not know the objective belief (as this require precise knowledge of the nature of traders' belief distortions). In recent work, Brunnermeier, Simsek, and Xiong (2012), we propose a welfare criterion that circumvents this difficulty. In particular, we take any convex combination of agents' beliefs as a reasonable objective belief, and we say that an allocation is *belief-neutral inefficient* if it is inefficient according to *all reasonable beliefs*.

The belief-neutral criterion provides a clear welfare ranking in this model. To see this, let $\mathbf{h}$, with $h_i \geq 0$ and $\sum_i h_i = 1$, denote an arbitrary convex combination of agents' beliefs. In particular, the distribution of $\mathbf{v}$ according to belief $\mathbf{h}$ is given by $N\left(\boldsymbol{\mu}_{\mathbf{h}}^{\mathbf{v}}, \Lambda^{\mathbf{v}}\right)$, where $\mu_{\mathbf{h}}^{\mathbf{v}} = \sum_i h_i \boldsymbol{\mu}_i^{\mathbf{v}}$. Consider the sum of traders' certainty equivalent net worths under belief $\mathbf{h}$, given by:

$$N_{\mathbf{h}} = \sum_{i \in I} \left(E_{\mathbf{h}}\left[n_i\right] - \frac{\theta_i}{2} var_{\mathbf{h}}\left(n_i\right) \right).$$

This expression is a measure of welfare under belief $\mathbf{h}$, in the sense that any allocation $\tilde{\mathbf{x}}$ that

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[14]In recent work, Gilboa, Samuelson, and Schmeidler (2012) argue that the Pareto criterion is not appropriate even under this interpretation, and offer an alternative welfare criterion.

yields a higher $N_{\mathbf{h}}$ than another allocation $\mathbf{x}$ can also be made to Pareto dominate allocation $\mathbf{x}$ (under belief $\mathbf{h}$) after combining it with appropriate ex-ante wealth transfers. Using the expressions (1) and (12), along with the market clearing condition $\sum_{i\in I}\mathbf{x}_i = 0$, this welfare measure can further be simplified to:

$$N_{\mathbf{h}} = E_{\mathbf{h}}\left[\sum_{i\in I} e + w_i\right] - \frac{\bar{\theta}}{2}\Omega.$$

The first component of welfare is agents' expected endowment, which is exogenous in the sense that it does not depend on agents' portfolios. The second and the endogenous component is proportional to average variance, Ω. Importantly, this component is independent of belief $\mathbf{h}$, illustrating the belief-neutral nature of the welfare analysis. Intuitively, trading in this economy does not generate expected net worth since it simply redistributes wealth. Hence, trading affects social welfare only through portfolio variances. Since there is agreement about variances [cf. assumption (A1)], social welfare can be analyzed without taking a stance on whose belief is correct.[15]

It follows that the average variance, Ω, emerges as a belief-neutral welfare measure. Consequently, the equilibrium is belief-neutral Pareto inefficient as long as the average variance deviates from its minimum, $\Omega > \Omega^R$. However, it might be difficult for the planner to implement the minimum, Ω^R, as this would require monitoring that each trader holds exactly the risk sharing portfolio, $\mathbf{x}_i^R$. Realistically, the planner might have to decide whether or not to allow unrestricted trade in new assets. A planner subject to this restriction would conclude that financial innovation leads to a belief-neutral inefficient allocation as long as new assets increase the average variance. More specifically, the equilibrium with new assets is belief-neutral Pareto dominated by the equilibrium without the new assets (plus appropriate ex-ante wealth transfers) if and only if financial innovation increases Ω.

6.2 Inefficiencies Driven by Externalities

The welfare implications of the baseline setting can also be overturned if traders' choices are associated with externalities. Such externalities naturally emerge when traders are viewed as financial intermediaries. In particular, these intermediaries might take socially excessive risks either because of fire sale externalities (see, for example, Lorenzoni (2008) and Stein (2011)), or because they enjoy explicit or implicit government protection (see Rajan (2010) for a discussion in the context of the recent crisis). Perhaps for these reasons, much existing regulation in the financial system is concerned with restricting intermediaries' portfolio risks. To the extent that financial innovation increases these risks, it could naturally lead to inefficiencies.

I next illustrate these types of inefficiencies in a version of the model in which traders are under government protection. Suppose each trader, i, corresponds to a financial intermediary

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[15] It is easy to see that this analysis can also be generalized to cases in which traders have reasonable disagreements about variances.

with limited liability. An intermediary whose net worth falls below zero, $n_i < 0$, is forced into bankruptcy, and its creditors potentially face losses. However, there is a new agent, the government, which bails out the creditors of an intermediary that enters bankruptcy (for reasons that are outside this model). For simplicity, suppose the government makes the creditors whole by paying $-n_i > 0$. In addition, the government inflicts a non-pecuniary punishment to the bankrupt intermediary that is equivalent to a pecuniary loss of $-n_i$ dollars. This additional assumption considerably simplifies the analysis by ensuring that the intermediary's problem (2) remains unchanged despite the limited liability feature. It is also a conservative assumption in the sense that it eliminates the additional portfolio risks that would stem from the usual risk-shifting motive, and enables me to focus on speculative risks.

The equilibrium characterization in this setting is the same as in Section 3. However, the welfare analysis is potentially different since the government is also affected by the intermediaries' portfolio choices. Suppose the government is risk-neutral and its belief for the underlying uncertainty is given by $N\left(\boldsymbol{\mu}_g^{\mathbf{v}}, \Lambda^{\mathbf{v}}\right)$. The government's welfare is then inversely proportional to its expected losses:

$$L_g = -\sum_{i=1}^{N} \Pr_g\left(n_i < 0\right) E_g\left[n_i \mid n_i < 0\right].$$

In particular, the government's losses, L_g, depend on a measure of intermediaries' average portfolio risks that is similar to (although not the same as) the average variance measure, Ω [cf. Eq. (12)]. Intuitively, average portfolio risks matter because they determine the extent to which financial intermediaries will need government assistance. Since financial innovation can increase these risks, it can also increase, L_g, illustrating the negative externalities. Note also that financial innovation will be inefficient, even in the usual Pareto sense, as long as these externalities, $L_g\left(J\right) - L_g\left(J_O\right)$, exceed the intermediaries' perceived private benefits.[16] The analysis in the appendix establishes sufficient conditions for Pareto inefficiency in the context of the example analyzed in Section 2.1, and illustrates that this outcome is more likely when the intermediaries' initial balance sheets are weaker.

7   Conclusion

This paper analyzed the effect of financial innovation on portfolio risks in a standard mean-variance setting with belief disagreements. In this framework, I defined the average variance of traders' net worths as a natural measure of portfolio risks. I also decomposed the average variance into two components: the uninsurable variance, defined as the variance that would obtain if there were no belief disagreements, and the speculative variance, defined as the residual amount of variance that results from speculation based on belief disagreements. My main result characterized the effect of financial innovation on both components of the average

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[16]These benefits can be explicitly calculated as $\sum_{i \in I} \pi_i\left(J\right) - \sum_{i \in I} \pi_i\left(J_O\right)$, where $\pi_i\left(\hat{J}\right)$ denotes the intermediary's willingness to pay to trade the assets in $\hat{J}$ [cf. Eq. (19)].

variance. Financial innovation always reduces the uninsurable variance through the traditional channels of diversification and the efficient transfer of risks. However, financial innovation also *always* increases the speculative variance, through two distinct economic channels. First, new assets generate new disagreements. Second, new assets amplify traders' speculation on existing disagreements. The increase in speculative variance may be sufficiently large that financial innovation may *increase* portfolio risks. I also showed that *endogenous financial innovation* is at least partly driven by the speculation motive for trade. When disagreements are sufficiently large, a profit seeking market maker innovates assets that *maximize* the average variance among all possible choices, completely disregarding the risk sharing motive in financial innovation.

These results show that belief disagreements can potentially overturn a number of traditional views regarding the relationship between financial innovation and portfolio risks. A natural question is how large belief disagreements should be for this analysis to be practically relevant. In Simsek (2011), I address this question by considering a calibration of the model in the context of the national income markets proposed by Shiller (1993). These assets could in principle facilitate the sharing of income risks among different countries. Athanasoulis and Shiller (2001) analyze these assets in the context of G7 countries, and argue that they would considerably increase welfare by reducing individuals' consumption risks. Using exactly their data and calibration, I find that small amounts of belief disagreements about the GDP growth rates of G7 countries (much smaller than implied by Philadelphia Fed's Survey of Professional Forecasters) imply that these assets would actually increase average consumption risks. While this calibration exercise is promising, it is far from conclusive. I leave empirical analysis of the model for future research.

Another natural question concerns the welfare implications of these results. In the baseline setting, financial innovation results in a Pareto improvement despite the fact that it might increase portfolio risks. However, I showed that this welfare conclusion can be overturned when belief disagreements are driven by psychological distortions, in which case the Pareto criterion is not appropriate; or when traders are viewed as financial intermediaries whose portfolio choices represent negative externalities. In either case, a measure of average portfolio risks emerge as a central object in welfare analysis, providing some normative content to Theorems 1 and 2. That said, I do not take a strong normative stance in this paper since the model is missing some important ingredients that could change the welfare arithmetic. Most importantly, speculation driven by financial innovation could provide some additional social benefits by making asset prices more informative. Assessing the net welfare effect of financial innovation is a fascinating question which I leave for future work.

A Appendix: Omitted Results and Proofs

Proof of Lemma 1. Recall that the objective function for Problem (13) is given by

$$\Omega = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \left(\mathbf{W}_i' \Lambda^{\mathbf{v}} \mathbf{W}_i + \mathbf{x}_i' \Lambda \mathbf{x}_i + 2\mathbf{x}_i' \boldsymbol{\lambda}_i \right). \tag{A.1}$$

The first order conditions are given by:

$$\Lambda \mathbf{x}_i + \boldsymbol{\lambda}_i = \boldsymbol{\gamma} \frac{\bar{\theta}}{\theta_i} \text{ for each } i \in I,$$

where $\boldsymbol{\gamma} \in \mathbb{R}^{|J_E|}$ is a vector of Lagrange multipliers. Note that $\mathbf{x}_i^R = -\Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i$ satisfies these first order conditions for the Lagrange multiplier $\boldsymbol{\gamma} = \left(\sum_{i \in I} \boldsymbol{\lambda}_i \right) / |I|$. This shows that $\left\{ \mathbf{x}_i^R \right\}_i$ is the unique solution to Problem (13).

Proof of Lemma 2. Plugging in $\mathbf{x}_i^R = -\Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i$ into the objective function $(A.1)$, the optimal value, Ω^R, is given by:

$$
\begin{aligned}
\Omega^R &= \frac{1}{|I|} \sum_i \frac{\theta_i}{\bar{\theta}} \left(\mathbf{W}_i' \Lambda^{\mathbf{v}} \mathbf{W}_i + \tilde{\boldsymbol{\lambda}}_i' \Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i \right) - \frac{2}{|I|} \sum_i \tilde{\boldsymbol{\lambda}}_i' \Lambda^{-1} \frac{\theta_i \boldsymbol{\lambda}_i}{\bar{\theta}}, \\
&= \frac{1}{|I|} \sum_i \frac{\theta_i}{\bar{\theta}} \left(\mathbf{W}_i' \Lambda^{\mathbf{v}} \mathbf{W}_i + \tilde{\boldsymbol{\lambda}}_i' \Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i \right) - \frac{2}{|I|} \sum_i \tilde{\boldsymbol{\lambda}}_i' \Lambda^{-1} \frac{\theta_i \tilde{\boldsymbol{\lambda}}_i}{\bar{\theta}} \\
&= \frac{1}{|I|} \sum_i \frac{\theta_i}{\bar{\theta}} \left(\mathbf{W}_i' \Lambda^{\mathbf{v}} \mathbf{W}_i - \tilde{\boldsymbol{\lambda}}_i' \Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i \right).
\end{aligned}
$$

Here, the second line uses the fact that $\sum_i \tilde{\boldsymbol{\lambda}}_i = 0$ to replace $\frac{\theta_i \boldsymbol{\lambda}_i}{\bar{\theta}}$ with its deviation from average, $\frac{\theta_i \tilde{\boldsymbol{\lambda}}_i}{\bar{\theta}}$. This completes the derivation of Eq. (14).

To derive Eq. (15), first consider the expression $|I| \left(\Omega - \frac{1}{|I|} \sum_i \frac{\theta_i}{\bar{\theta}} \mathbf{W}_i' \Lambda^{\mathbf{v}} \mathbf{W}_i \right)$. Using the definition of the average variance in (12), this expression can be written as:

$$
\begin{aligned}
\sum_i \frac{\theta_i}{\bar{\theta}} \mathbf{x}_i' \Lambda \mathbf{x}_i + 2 \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \mathbf{x}_i' \boldsymbol{\lambda}_i &= \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i \right)' \Lambda^{-1} \left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i \right) + 2 \sum_{i \in I} \left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i \right)' \Lambda^{-1} \frac{\theta_i \boldsymbol{\lambda}_i}{\bar{\theta}} \\
&= \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \left[\left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i \right)' \Lambda^{-1} \left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i \right) + 2 \left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i \right)' \Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i \right] \\
&= \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \left[\left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i \right)' \Lambda^{-1} \left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} + \tilde{\boldsymbol{\lambda}}_i \right) \right] \\
&= \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \frac{\tilde{\boldsymbol{\mu}}_i'}{\theta_i} \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \tilde{\boldsymbol{\lambda}}_i' \Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i.
\end{aligned}
$$

Here, the first line substitutes for the portfolio demands from (9); the second line re-

places $\frac{\theta_i \lambda_i}{\bar{\theta}}$ with its deviation from average, $\frac{\theta_i \tilde{\lambda}_i}{\bar{\theta}}$ (as in the first part of the proof); and the next two lines follow by simple algebra. Next, using the fact that the last line equals $|I| \left(\Omega - \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \mathbf{W}_i' \Lambda^{\mathbf{v}} \mathbf{W}_i \right)$, the average variance can be written as:

$$\Omega = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \left(\mathbf{W}_i' \Lambda^{\mathbf{v}} \mathbf{W}_i - \tilde{\lambda}_i' \Lambda^{-1} \tilde{\lambda}_i \right) + \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \frac{\tilde{\boldsymbol{\mu}}_i'}{\theta_i} \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i}.$$

Using the definition of Ω^R in (14), it follows that the speculative variance is given by the expression in (15).

Proof of Theorem 1. **Part (i).** By definition, Ω^R, is the optimal value of the minimization problem (13). Financial innovation expands the constraint set of this problem. Thus, it also decreases the optimal value, proving $\Omega^R (J_O \cup J_N) \leq \Omega^R (J_O)$.

Part (ii). The proof proceeds in three steps. First, the form of $\mathbf{x}_i^S$ in Eq. (9) implies that the speculative portfolio, $\mathbf{x}_i^S$, solves the following version of the mean-variance problem:

$$\max_{\mathbf{x}_i \in \mathbb{R}^{\hat{J}}} (\tilde{\boldsymbol{\mu}}_i)' \mathbf{x}_i - \frac{\theta_i}{2} \mathbf{x}_i' \Lambda \mathbf{x}_i. \tag{A.2}$$

Moreover, the speculative variance, Ω^S, is found by averaging the variance costs for each trader at the solution to this problem:

$$\Omega^S = \frac{1}{|I|} \sum_i \frac{\theta_i}{\bar{\theta}} \left(\mathbf{x}_i^S \right)' \Lambda \mathbf{x}_i^S. \tag{A.3}$$

Intuitively, problem $(A.2)$ is the traders' mean-variance problem in a hypothetical economy that is identical except that traders have no background risks (i.e., $\mathbf{W}_i = 0$ for all $i \in I$), so that the only motive for trade is speculation. The solution to this problem gives the speculative portfolio in the actual economy, and also determines the speculative variance as captured by $(A.3)$.

Second, financial innovation relaxes the constraint set of problem $(A.2)$. That is, when the asset set is $\hat{J} = J^O \cup J^N$, traders are able to make all the speculative trades they could make when the asset set is $\hat{J} = J^O$, and some more. Intuitively, new assets expand the betting possibilities frontier through the two distinct channels emphasized in the main text. Third, when the constraint set of problem $(A.2)$ is more relaxed, each trader obtains a greater certainty-equivalent payoff from betting. Moreover, since the problem is a quadratic optimization, expected payoffs at the optimum are proportional to the expected variance of the payoffs, that is: $(\tilde{\boldsymbol{\mu}}_i)' \mathbf{x}_i^s = 2 \frac{\theta_i}{2} (\mathbf{x}_i^s)' \Lambda \mathbf{x}_i^s$. Consequently, a relaxed constraint set also increases the expected variance, $\frac{\theta_i}{2} (\mathbf{x}_i^s)' \Lambda \mathbf{x}_i^s$. Intuitively, at the optimal speculative portfolio, higher expected returns go hand-in-hand with higher risks. These steps establish that financial innovation increases the speculative variance of each trader. It follows that financial innovation

also increases the average speculative variance in Eq. $(A.3)$, completing the proof.[17]

Proof of Proposition 1. Using Eqs. (9),(15), and (14), the uninsurable and the speculative variance can be written in terms of traders' positions as:

$$\Omega^R = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \left(\mathbf{W}_i' \Lambda^{\mathbf{v}} \mathbf{W}_i - \left(\mathbf{x}_i^R \right)' \Lambda \mathbf{x}_i^R \right) \text{ and } \Omega^S = \frac{1}{|I|} \sum_i \frac{\theta_i}{\bar{\theta}} \left(\mathbf{x}_i^S \right)' \Lambda \mathbf{x}_i^S.$$

Eq. (18) then follow by considering respectively $\Omega^R(J_O) - \Omega^R(J)$ and $\Omega^S(J) - \Omega^R(J_O)$.

Proof of Proposition 2. Since asset j_N is uncorrelated with the remaining assets, the matrices Λ and Λ^{-1} are both block diagonal. Hence, Eq. (9) illustrates that the introduction of asset j_N does not affect the position on the remaining assets. Using this observation along with the previous expressions, the reduction in uninsurable portfolio risks can be written as:

$$\left| \Delta \Omega^R \right| = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\bar{\theta}} \left(\mathbf{x}_i^{j_N, R} \right)' \Lambda^{j_N j_N} \mathbf{x}_i^{j_N, R} = \Lambda^{j_N j_N} \left(T^{j_N, R} \right)^2.$$

Similarly, the increase in speculative portfolio risks is given by $\left| \Delta \Omega^S \right| = \Lambda^{j_N j_N} \left(T^{j_N, S} \right)^2$. The proof of the proposition follows from comparing these two expressions.

Derivation of the Market Maker's Profit in Section 5 First note that trader i's payoff from rejecting the market maker's offer is the certainty equivalent payoff from her endowment:

$$e + \mathbf{W}_i' \boldsymbol{\mu}_i^{\mathbf{v}} - \frac{\theta_i}{2} \mathbf{W}_i' \Lambda^{\mathbf{v}} \mathbf{W}_i. \tag{A.4}$$

Next consider trader i's certainty equivalent payoff after trading the assets. Using Eq. (9), traders' net worth, n_i, can be written as:

$$n_i = e - \mathbf{x}_i' \mathbf{p} + \left[\mathbf{W}_i + \mathbf{A} \Lambda^{-1} \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A}) \right) \right]' \mathbf{v}.$$

The certainty equivalent of this expression is given by:

$$e - \mathbf{x}_i' \mathbf{p} + \mathbf{W}_i' \boldsymbol{\mu}_i^{\mathbf{v}} + \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A}) \right)' \Lambda^{-1} \boldsymbol{\mu}_i \tag{A.5}$$

$$- \frac{\theta_i}{2} \mathbf{W}_i' \Lambda^{\mathbf{v}} \mathbf{W}_i - \frac{\theta_i}{2} \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A}) \right)' \Lambda^{-1} \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A}) \right) - \theta_i \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A}) \right) \Lambda^{-1} \boldsymbol{\lambda}_i(\mathbf{A}).$$

Since the fixed fee makes the trader indifferent, it is equal to the difference of the expression in $(A.5)$ from the expression in $(A.4)$. That is:

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[17]See the working paper version (same title, NBER working paper No. 17506) for an alternative proof based on matrix algebra.

$$\pi_i\left(\mathbf{A}\right) = -\frac{\theta_i}{2}\left(\frac{\tilde{\boldsymbol{\mu}}_i\left(\mathbf{A}\right)}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i\left(\mathbf{A}\right)\right)'\left(\frac{\tilde{\boldsymbol{\mu}}_i\left(\mathbf{A}\right)}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i\left(\mathbf{A}\right)\right)$$
$$-\mathbf{x}_i'\mathbf{p} + \left(\frac{\tilde{\boldsymbol{\mu}}_i\left(\mathbf{A}\right)}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i\left(\mathbf{A}\right)\right)'\left(\boldsymbol{\mu}_i\left(\mathbf{A}\right) - \theta_i\boldsymbol{\lambda}_i\left(\mathbf{A}\right)\right). \tag{A.6}$$

Next consider the sum of the fixed fees over all i. Note that $\sum_i -\mathbf{x}_i'\mathbf{p} = \mathbf{0}$ in view of market clearing. Using the definitions in (10) and (11), note also that $\sum \frac{\tilde{\mu}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i\left(\mathbf{A}\right) = 0$. This further implies

$$\sum_i \left(\frac{\tilde{\boldsymbol{\mu}}_i\left(\mathbf{A}\right)}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i\left(\mathbf{A}\right)\right)'\left(\boldsymbol{\mu}_i\left(\mathbf{A}\right) - \theta_i\boldsymbol{\lambda}_i\left(\mathbf{A}\right)\right) = \sum_i \left(\frac{\tilde{\boldsymbol{\mu}}_i\left(\mathbf{A}\right)}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i\left(\mathbf{A}\right)\right)'\left(\tilde{\boldsymbol{\mu}}_i\left(\mathbf{A}\right) - \theta_i\tilde{\boldsymbol{\lambda}}_i\left(\mathbf{A}\right)\right).$$

Using these observations, the sum of the fixed fees in $(A.6)$ is given by (19), completing the derivation.

Proof of Proposition 3. To prove the result it is useful to consider the market maker's optimization problem in terms of a linear transformation of assets, $\hat{\mathbf{A}} = (\Lambda^{\mathbf{v}})^{1/2}\mathbf{A}$, where $(\Lambda^{\mathbf{v}})^{1/2}$ is the unique positive definite square root matrix of $\Lambda^{\mathbf{v}}$. Note that choosing $\hat{\mathbf{A}}$ is equivalent to choosing $\mathbf{A}$. The normalizations in (20) can be written in terms of $\hat{\mathbf{A}}$ as:

$$\hat{\mathbf{A}}'\hat{\mathbf{A}} = \mathbf{I}_{|J|}, \qquad \text{and} \qquad \tilde{\mathbf{A}}_1^j \geq 0 \text{ for each } j. \tag{A.7}$$

After using the normalization $\Lambda = \mathbf{I}_{|J|}$ and substituting $\hat{\mathbf{A}}$ for $\mathbf{A}$, the expected profit in (19) can also be written as:

$$\sum_i \pi_i\left(\hat{\mathbf{A}}\right) = \sum_i \frac{\theta_i}{2}\left((\Lambda^{\mathbf{v}})^{-1/2}\frac{\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2}\tilde{\mathbf{W}}_i\right)'\hat{\mathbf{A}}\hat{\mathbf{A}}'\left((\Lambda^{\mathbf{v}})^{-1/2}\frac{\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2}\tilde{\mathbf{W}}_i\right),$$
$$= tr\left(\hat{\mathbf{A}}'\mathbf{M}\hat{\mathbf{A}}\right) = \sum_j \left(\hat{\mathbf{A}}^j\right)'\mathbf{M}\hat{\mathbf{A}}^j,$$
$$\text{where } \mathbf{M} = \left((\Lambda^{\mathbf{v}})^{-1/2}\frac{\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2}\tilde{\mathbf{W}}_i\right)\left((\Lambda^{\mathbf{v}})^{-1/2}\frac{\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2}\tilde{\mathbf{W}}_i\right)'. \tag{A.8}$$

Here, the second line uses the matrix identity $tr\left(XY\right) = tr\left(YX\right)$ and the linearity of the trace operator, and the last line defines the $m \times m$ matrix, $\mathbf{M}$. Thus, the market maker's problem reduces to choosing $\hat{\mathbf{A}} = (\Lambda^{\mathbf{v}})^{1/2}\mathbf{A}$ to maximize $(A.8)$ subject to the normalizations in $(A.7)$.

Next note that the first normalization in (20) implies:

$$\left(\hat{\mathbf{A}}^j\right)'\hat{\mathbf{A}}^j = 1 \text{ for each } j. \tag{A.9}$$

Consider the alternative problem of choosing $\hat{\mathbf{A}}$ to maximize the expression in $(A.8)$ subject

to the relaxed constraint in $(A.9)$. The first order conditions for this problem are given by

$$\mathbf{M}\hat{\mathbf{A}} = \gamma^j \hat{\mathbf{A}}^j \text{ for each } j,$$

where $\gamma^j \in \mathbb{R}_+$ are Lagrange multipliers. From this expression, it follows that $\left\{\hat{\mathbf{A}}^j\right\}_j$ correspond to eigenvectors of the matrix, $\mathbf{M}$, and $\left\{\gamma^j\right\}_j$ correspond to eigenvalues. Plugging the first order condition into Eq. $(A.8)$, the expected profit can be written as:

$$\sum_i \pi_i\left(\hat{\mathbf{A}}\right) = \sum_j \gamma^j \left(\hat{\mathbf{A}}^j\right)' \hat{\mathbf{A}}^j = \sum_j \gamma^j.$$

It follows that the objective value will be maximized if and only if $\left\{\gamma^j\right\}_j$ correspond to the $|J|$ largest eigenvalues of the matrix, $\mathbf{M}$. If the $|J|$ largest eigenvalues are unique, then the optimum vectors, $\hat{\mathbf{A}} = \left\{\hat{\mathbf{A}}^j\right\}_j$, are uniquely characterized as the corresponding eigenvectors which have length 1 [cf. Eq. $(A.9)$] and which satisfy the sign convention in $(A.7)$. If the $|J|$ largest eigenvalues are not unique, then the same argument shows that the vectors, $\left\{\hat{\mathbf{A}}^j\right\}_j$, are uniquely determined up to a choice of these eigenvalues.

Finally, consider the original problem of maximizing the expression in $(A.8)$ subject to the stronger condition, $\hat{\mathbf{A}}'\hat{\mathbf{A}} = \mathbf{I}$. Since $\mathbf{M}$ is a symmetric matrix, its eigenvectors are orthogonal. This implies that the solution, $\left\{\hat{\mathbf{A}}^j\right\}_j$, to the alternative problem is in the constraint set of the original problem. Since the latter problem has a stronger constraint, it follows that the solutions to the two problems are the same, completing the proof.

Proof of Theorem 2. **Part (i).** Note that $\tilde{\boldsymbol{\mu}}_{i,0}\left(\mathbf{A}\right) = \mathbf{A}'\tilde{\boldsymbol{\mu}}_{i,0}^{\mathbf{Y}} = 0$ for any $\mathbf{A}$. This implies that the expected profit in (19) is given by $\sum_i \frac{\theta_i}{2}\tilde{\boldsymbol{\lambda}}_{\mathbf{i}}\left(\mathbf{A}\right)'\Lambda^{-1}\tilde{\boldsymbol{\lambda}}_{\mathbf{i}}\left(\mathbf{A}\right)$. From Eq. (14), this expression is equal to $c_1 - c_2\Omega^R\left(\mathbf{A}\right)$ for some constant c_1 and positive constant c_2. Thus, maximizing $\sum_i \pi_i\left(\mathbf{A}\right)$ is equivalent to minimizing $\Omega^R\left(\mathbf{A}\right)$. Finally, note from Eq. (15), that $\Omega_0^S\left(\mathbf{A}\right) = 0$ for any $\mathbf{A}$. This further implies $\Omega\left(\mathbf{A}\right) = \Omega^R\left(\mathbf{A}\right)$, proving that the market maker innovates assets that minimize $\Omega\left(\mathbf{A}\right)$.

Part (ii). Consider the following objective function:

$$\frac{1}{K^2}\sum_i \pi_{i,K}\left(\mathbf{A}\right), \tag{A.10}$$

which is just a scaling of the expected profit in (19). In particular, maximizing this expression is equivalent to maximizing the expected profit. In view of Proposition 3, the optimal asset design, $\mathbf{A}_K$, is uniquely determined. This also implies that $\mathbf{A}_K$ is a continuous function. Since $\mathbf{A}_K$ is bounded [from the normalization (20)], it follows that $\lim_{K\to\infty}\mathbf{A}_K$ exists.

Note also that the limit of the objective function in $(A.10)$ can be calculated as:

$$\lim_{K \to \infty} \frac{1}{K^2} \sum_i \pi_{i,K}(\mathbf{A}) = \lim_{K \to \infty} \sum_i \frac{\boldsymbol{\theta}_i}{2} \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \frac{\tilde{\boldsymbol{\lambda}}_i(\mathbf{A})}{K} \right)' \Lambda^{-1} \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \frac{\tilde{\boldsymbol{\lambda}}_i(\mathbf{A})}{K} \right)$$
$$= \sum_i \frac{\theta_i}{2} \frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})'}{\theta_i} \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i}, \tag{A.11}$$

where the first line uses $\tilde{\boldsymbol{\mu}}_{i,K}(\mathbf{A}) = K\tilde{\boldsymbol{\mu}}_i(\mathbf{A})$ and the second line uses $\tilde{\boldsymbol{\mu}}_i(\mathbf{A}) = \mathbf{A}'\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}$. In particular, the objective function remains bounded as $K \to \infty$. Thus, Berge's Maximum Theorem applies and implies that $\mathbf{A}_K$ is upper hemicontinuous in K over the extended set $\mathbb{R}_+ \cup \{\infty\}$. In particular, $\lim_{K \to \infty} \mathbf{A}_K$ maximizes the limit objective function in $(A.11)$ subject to the normalization, (20).

Finally, consider the limit of the average variance

$$\lim_{K \to \infty} \frac{\Omega_K\left(\hat{\mathbf{A}}\right)}{K^2} = \lim_{K \to \infty} \left(\frac{\Omega_K^S\left(\hat{\mathbf{A}}\right)}{K^2} + \frac{\Omega_K^R\left(\hat{\mathbf{A}}\right)}{K^2} \right) = \frac{1}{|I|} \sum_i \frac{\theta_i}{\bar{\theta}} \frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})'}{\theta_i} \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i},$$

where the second equality follows from Eqs. (15) and (14). In view of Eq. $(A.11)$, it follows that $\lim_{K \to \infty} \mathbf{A}_K$ maximizes $\lim_{K \to \infty} \frac{1}{K^2} \Omega_K\left(\hat{\mathbf{A}}\right)$ subject to the normalization, (20), completing the proof.

Sufficient conditions for Pareto inefficiency in Section 6.2. Consider the environment in Section 2.1 with the only difference that $w_1 = w_2 = 0$, so that the only reason for trade is speculation (which keeps the analysis simple). Suppose both assets 1 and 2 are introduced for trade. Without financial innovation, the intermediaries' net worths are constant, i.e., $n_1 = n_2 = e$, and the government's expected loss, L_g, is zero. Financial innovation increases portfolio risks [cf. Eq. (7)], which leads to $L_g > 0$. To calculate losses explicitly, suppose the government's belief is the average of the two intermediaries' beliefs, i.e., $v_1 \sim_g N(0, 1)$. This implies:

$$L_g = 2\frac{\varepsilon}{\theta}\left(\phi(\eta) - \eta\boldsymbol{\phi}(-\eta)\right) > 0, \text{ where } \eta = \frac{e}{\varepsilon/\theta}.$$

Here, $\phi(\cdot)$ and $\boldsymbol{\phi}(\cdot)$ are respectively the pdf and the cdf of the standard normal distribution and η is the standardized mean of the intermediary's net worth [cf. Eq. (7)]. The function, $\phi(\eta) - \eta\boldsymbol{\phi}(-\eta)$, is decreasing, illustrating the negative relationship between the government's losses and the strength of the intermediary's balance sheet. Using Eq. (19), the intermediaries' total private benefit can also be calculated as $\frac{\varepsilon^2}{\theta}$. Combining these observations, financial innovation is Pareto inefficient as long as:

$$\varepsilon < 2\left(\phi(\eta) - \eta\boldsymbol{\phi}(-\eta)\right).$$

This condition is satisfied when e and ε are sufficiently small (in particular, it suffices to take $e \sim 0$ and $\varepsilon < 0.79$). Intuitively, a small e (weaker initial balance sheets) ensures that the negative externalities are relatively large, whereas a small ε ensures that private gains from speculation are relatively small.

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